

# The Fragile Benefits of Endowment Destruction

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## Abstract

We show that the benefits of endowment destruction documented by Ljungqvist and Uhlig (2014), and the related point that rising consumption can lower habits, are fragile results of a particular discretization. Discretizing more frequently, both results disappear. One way of extending the continuous-time version of the model to jumps produces the puzzles, but arbitrarily close continuous paths or other ways of extending the model to jumps do not produce the result. Nonetheless, this example provides a useful lesson of how to approach apparently strange predictions of calibrated models.

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# 1 Endowment Destruction and the Effect of Consumption on Habit

Suppose the log endowment at time 1 is zero,  $y_1 = 0$ . Suppose the government destroys some of this endowment, so that consumption  $c_1 = \psi < 0$ . For all other periods, consumption equals the endowment,  $c_t = y_t$ . Now, endowment destruction at time zero will hurt time zero utility. But it can lower the habit for future periods, potentially leaving the consumer better off. (Endowment destruction only lasts one period; permanently lower consumption is harmful.)

To evaluate the effect of endowment destruction, suppose a Campbell-Cochrane (1999) consumer at time  $t = 0$  is at the steady-state log surplus consumption ratio  $s_0 = \bar{s}$ . Suppose log endowment grows steadily at the rate  $g$  for periods 0, 1, 2, i. e.  $y_0 = -g$ ,  $y_1 = 0$ ,  $y_2 = g$ , and thereafter the log endowment follows the usual process  $y_{t+1} = g + y_t + v_{t+1}$ ,  $t > 2$ .

The solid line in Figure 1 presents the consumer’s welfare in this situation, using a monthly time interval, for a variety of  $\psi$ . We include  $\psi > 0$ , transfers from abroad or manna from Heaven. Near  $\psi = 0$ , welfare rises with  $\psi$ . Despite the habit formation, output destruction hurts and transfers help. However, destroying a discrete amount of monthly output raises welfare. This is Ljungqvist and Uhlig’s (2014) main point.

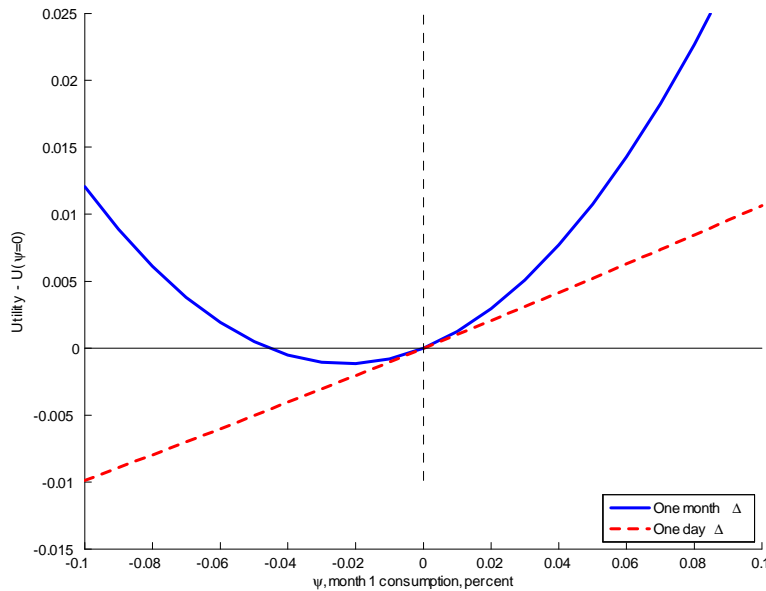


Figure 1: Effect of endowment destruction or transfers. At time  $t = 1$ , an amount  $\psi$  is subtracted or added to log consumption. The Figure plots the achieved welfare as a function of  $\psi$ . The solid line uses a monthly discretization, and perturbs consumption at time  $t = 1$  only. The dashed line uses a daily discretization, modifying consumption in a V shaped pattern for two months.

In the dashed line of Figure 1, we use instead a daily discretization to simulate the model. Starting at time  $t = 0$  as before, we add (or subtract, when  $\psi < 0$ )  $\psi/30$  from log consumption each day for 30 days, and then restore consumption the same way, giving a V-shaped daily pattern that bottoms out at the same  $\psi$  value on the 30th day. In this daily discretization, the Ljungqvist and Uhlig pattern disappears. Output destruction is always harmful, and transfers always welcome. More frequent discretization leads to visually indistinguishable results, so one can regard this answer also as an approximate solution of the underlying continuous-time model.

(To create Figure 1, we follow the indicated first three months of consumption by a simulation of the endowment process. Then we evaluate the utility function by averaging over a large number of simulations.)

Why is there such a difference between the monthly and daily discretization of the model? Figure 2 explains. The top row gives the monthly discretization and the bottom row gives the daily discretization. The left column gives consumption and the right column gives the implied path of habit.

Top row: When consumption declines at time 1 (left, solid), habit declines (right, solid). But when consumption rises back to where it started, at time 2, habit declines again, and even more than the initial decline. This latter further decline in habit accounts for the benefits of endowment destruction. In a similarly puzzling fashion, when consumption rises at time 1 (left, dashed), habit declines at time 1 (right, dashed). In this case however, the puzzling behavior of habit just makes transfers even more valuable.

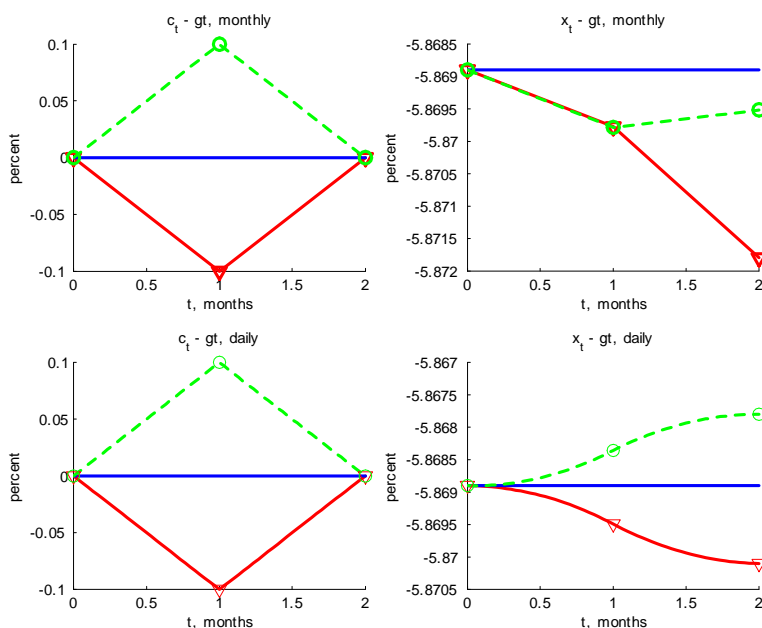


Figure 2: Paths of log consumption and log habit in the endowment destruction / transfer experiment. The top rows are the Ljungqvist and Uhlig monthly discretization, the bottom rows are the daily discretization. The growth rate  $gt$  is subtracted from all series for clarity. The left columns show the path of log consumption. The right columns show the path of log habit that results.  $\psi = \pm 0.1\%$ .

Bottom row: The bottom left panel shows the daily path of consumption in the destruction episode, in a daily discretization. This is not a repetition of the top left panel, as now there is a point for every day. In the bottom right panel, we see that habits rise with the consumption rise in the first month, and decline with the consumption decline, as they should. In the second month, habits continue to decline in the destruction path, but the decline is slower – habits are still adjusting smoothly to the first month decline, but the less severe second month decline is pushing them down less. Thus the consumer is left at the end of the second month of habit destruction with somewhat lower habit, but not enough to overcome the pain of losing the month’s consumption.

As Figure 2 shows, the benefits of endowment destruction are tied to the apparently strange fact that discrete changes in consumption can induce habits to move the other way, even though marginal changes in consumption always move habits in the same direction.

Figure 3 explores this mechanism in more detail, by presenting the relationship between monthly consumption growth and contemporaneous habit. (This graph presents the calculation when the surplus consumption ratio is at its steady state, which is the hardest case. The derivative  $dx_t/dc_t = 0$  in this case, represented by the slope of the solid line where it intersects the vertical line at  $\Delta c_t = g$ . For other values of the initial surplus consumption ratio, we have  $dx_t/dc_t > 0$ ; the solid curve moves to the right leaving a positive derivative in the middle.) The solid line verifies Ljungqvist and Uhlig's result, which we saw in Figure 2: Discrete increases in consumption can lead to a contemporaneous decline in habits.

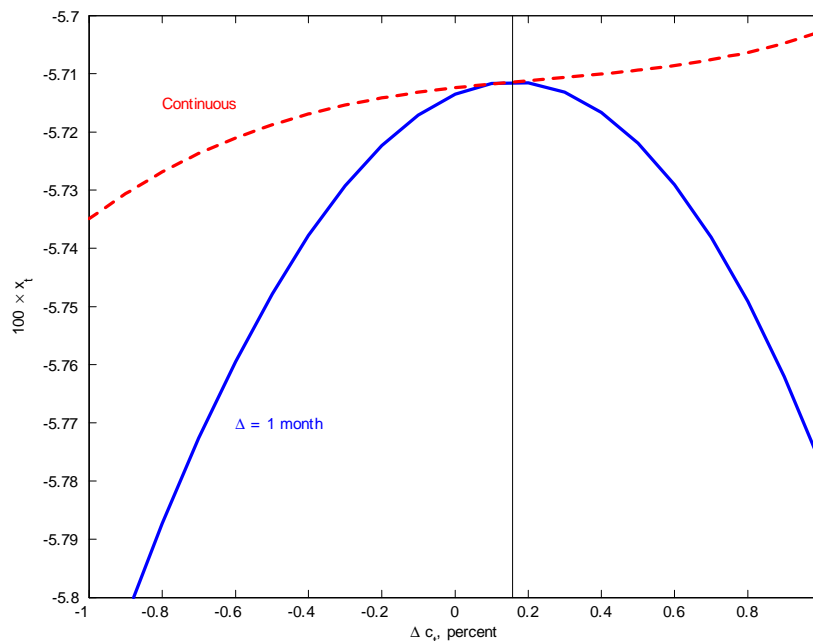


Figure 3: The effect of log consumption growth  $\Delta c_{t+1}$  on contemporaneous log habit  $x_{t+1}$ , when  $s_t = \bar{s}$ . The solid line uses a monthly horizon. The dashed line subdivides the consumption change into 30 steps. The vertical line indicates the value of consumption  $c_{t+1} - c_t = g$  at which the term multiplying  $\lambda(s_t)$  is zero in the surplus consumption ratio transition equation.

The dashed line in Figure 3 once again subdivides the monthly change in consumption into 30 increments. The figure is also visually identical for any finer discretization, so this can also be read as the continuous-time calculation with a linear change in consumption over a month. We see that in this version, habit is a nondecreasing function of consumption throughout.

In the monthly simulation, endowment destruction lowers habit, moving to the left in Figure 3. But then, the positive growth rate of consumption as it recovers from the habit episode, a move from the center to the right in Figure 3 lowers habit some more. (To be precise, that move happens in a version of Figure 3 recalculated at the new slightly lower surplus-consumption ratio, which is almost but not exactly the same.) It is the strong concavity of this function, really, which drives the result. And that concavity is absent in the finer discretization.

Why is the daily discretization so different from the monthly discretization? Examine the evo-

lution of the surplus consumption ratio,

$$\begin{aligned} s_{t+\Delta} &= (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+\Delta} - c_t - g\Delta); \\ \lambda(s) &= \frac{1}{\bar{S}}\sqrt{1 - 2(s - \bar{s})} - 1; \quad s \leq s_{\max} \\ \lambda(s) &= 0; \quad s \geq s_{\max} \end{aligned} \tag{1}$$

where

$$s_t = \log\left(\frac{C_t - X_t}{C_t}\right),$$

$c_t = \log C_t$  is consumption,  $X_t$  is habit,  $\bar{s} = \log \bar{S}$ ,  $\phi$ ,  $g$ ,  $s_{\max}$  are parameters, and  $\Delta$  is the discretization interval. During the daily discretization, the surplus consumption ratio responds to each little bit of consumption each day, and a new  $\lambda(s_t)$  is recomputed each day. During the monthly discretization the same  $\lambda(s_0)$  applies to all the changes from  $t = 0$  to  $t = 1$ , and the same  $\lambda(s_1)$  applies to all the changes from 1 to 2. A daily discretization that uses the beginning-of-the-month value of  $\lambda(s_t)$  rather than the continuously evolving one would generate Ljungqvist and Uhlig's result.

## 2 Continuous Time

Ljungqvist and Uhlig might respond, let the government destroy endowment for one day only, restoring it the next day. And we might respond, well, let it destroy the endowment a bit by bit each hour, and the effect again disappears.

You can see where this is going. Like all issues involving discretization, these issues are best understood by writing the underlying continuous-time version of our model, and considering different possibilities for its discretization or approximation. In continuous time, the surplus consumption ratio evolves as

$$ds_t = (1 - \phi)(\bar{s} - s_t)dt + \lambda(s_t)(dc_t - gdt); \tag{2}$$

the endowment follows a geometric Brownian motion

$$dc_t = gdt + \sigma dz_t;$$

and expected utility is

$$W = E \int_{t=0}^{\infty} e^{-\delta t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} dt = E \int_{t=0}^{\infty} \frac{e^{-\delta t} e^{(1-\gamma)(c_t + s_t)}}{1-\gamma} dt.$$

In this context, to produce Ljungqvist and Uhlig's calculation, one extends the model to consider jumps in the endowment. One specifies a downward jump  $dc_0 = \psi < 0$ , followed quickly by an upward jump at time  $\varepsilon$ ,  $dc_\varepsilon = -\psi + g\varepsilon$ . In extending the model to jumps to produce the endowment destruction result, one specifies that the  $\lambda(s_t^-)$ , with  $s_t^-$  equal to the value just before the jump, applies to the entire jump episode. Then, denoting values just before the jump as  $c_0^-$ ,  $s_0^-$ , etc., the surplus consumption ratio changes by

$$s_0 - s_0^- = \lambda(s_0^-)\psi.$$

On the way back up, however, and for  $\varepsilon$  small so we can ignore the  $dt$  term,

$$s_\varepsilon - s_\varepsilon^- = -\lambda(s_0)\psi = -\lambda(s_0)\psi.$$

Thus, the round trip produces a net change in the surplus consumption ratio,

$$s_\varepsilon - s_0^- = [\lambda(s_0^-) - \lambda(s_0)] \psi.$$

The slope of the  $\lambda(s)$  function ( $\lambda'(s) < 0$ ) means that the surplus consumption ratio rises at  $\varepsilon$  by more than it declined at 0, producing a net rise in the surplus consumption ratio, and thus a decline in the habit.

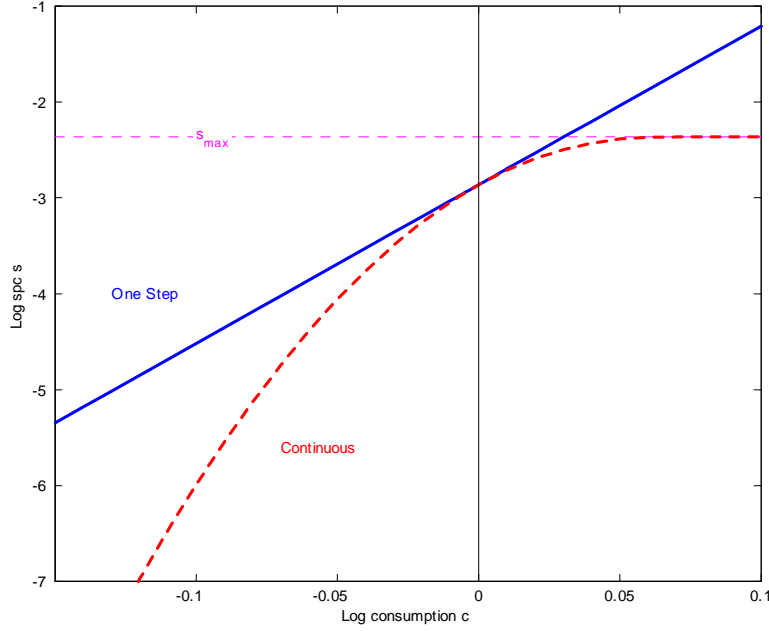


Figure 4: Log consumption and contemporaneous surplus consumption ratio. Starting at  $c_0 = 0$ ,  $s_0 = \bar{s}$ , the solid line plots  $s_1 = \phi(\bar{s} - s_0) + \lambda(s_0)(c_1 - c_0 - g)$  at a one-day horizon. The dashed line simulates  $s_{t+\Delta} = \phi(\bar{s} - s_t) + \lambda(s_t)(c_{t+\Delta} - c_t - g)$  in 300 subintervals of the day.

To illustrate, the solid line in Figure 4 plots the change in surplus consumption ratio implied by jumps in consumption, by  $s_0 - s_0^- = \lambda(s_0^-)(c_0 - c_0^-)$  with  $s_0^- = \bar{s}$  and  $c_0^- = 0$ . Since  $\lambda(s_0^-)$  is a constant, this is just a straight line. (To be precise, the Figure plots the discrete-time transition (1) at one-day interval  $\Delta$ .)

However, once again, suppose we produce the same decline and rise in consumption by a V shaped – or any rapid, but continuous – pattern. In this case, there is as  $\varepsilon \rightarrow 0$  no change at all in the surplus consumption ratio. And in any discretely-sampled data there is no way to tell the difference between the V shaped continuous decline and the jump.

To see how this works, the consumption path is now a steep V shape, which bottoms out at  $c_{\varepsilon/2} = \psi$

$$c_t = \frac{2\psi}{\varepsilon}t; \quad 0 < t < \frac{\varepsilon}{2};$$

$$c_t = -\frac{2\psi}{\varepsilon}\left(t - \frac{\varepsilon}{2}\right); \quad \frac{\varepsilon}{2} < t < \varepsilon.$$

Now, the surplus consumption ratio evolves as (2), and leaving off the small  $dt$  terms,

$$\begin{aligned} ds_t &= \lambda(s_t) \frac{2\psi}{\varepsilon} dt; \quad 0 < t < \frac{\varepsilon}{2}; \\ ds_t &= -\lambda(s_t) \frac{2\psi}{\varepsilon} dt; \quad \frac{\varepsilon}{2} < t < \varepsilon. \end{aligned}$$

The surplus consumption ratio at time  $\varepsilon/2$  and  $\varepsilon$  therefore solves the differential equation

$$\begin{aligned} \int_{s_0}^{s_{\varepsilon/2}} \frac{1}{\lambda(s)} ds &= \psi = c_{\varepsilon/2} - c_0; \\ \int_{s_{\varepsilon/2}}^{s_\varepsilon} \frac{1}{\lambda(s)} ds &= -\psi = -(c_\varepsilon - c_{\varepsilon/2}). \end{aligned} \tag{3}$$

The dashed curved line of Figure 4 clarifies the difference. Now, as consumption falls from time 0 to time  $\varepsilon/2$ , the surplus consumption ratio follows the dashed curved line, not the straight line. The dashed curved line expresses the solution of the differential equation (3). (To be precise, the Figure plots a 300 point discretization of the daily change, i.e. it solves the differential equation  $ds_t = -\phi(s_t - \bar{s}) dt + \lambda(s_t)(dc_t - gdt)$  over a day.)

The second differential equation in (3) exactly retraces the steps of the first one, so there is no change at all in the surplus consumption ratio when it is all over. In fact, the surplus consumption ratio returns to its original value for any continuous consumption path that returns to its initial value, for  $\phi = 1$  or in the  $\varepsilon \rightarrow 0$  limit.

Pathologically, then, a jump and instantaneous jump back produces a change in the surplus consumption ratio, but a continuous movement arbitrarily close to the jump produces no change at all in the surplus consumption ratio.

One can resolve this pathology quite simply, by extending the diffusion model to jumps in a different way. If jumps produce the changes in surplus consumption ratio described by the differential equation (3) and shown in the dashed line of Figure 4, rather than follow the linear function  $\lambda(s_0^-)(c_0 - c_0^-)$ , the jump and its approximating diffusion will produce the same result. To be specific, write the solutions to the differential equation

$$\int_{s^-}^s \frac{1}{\lambda(\sigma)} d\sigma = c - c^- \tag{4}$$

as

$$s - s^- = f(c - c^-, s^-)$$

Then, write the continuous-time model, extended to handle jumps, as

$$ds_t = (1 - \phi)(\bar{s} - s_t) dt + \lambda(s_t)(dc_t - gdt) + f(dJ_t, s_t^-)$$

rather than

$$ds_t = (1 - \phi)(\bar{s} - s_t) dt + \lambda(s_t)(dc_t - gdt) + \lambda(s_t^-)dJ_t$$

where  $dJ_t$  is a consumption jump. In essence, we specify that jumps in consumption follow the same the dashed curved line of Figure 4 that very rapid continuous movements follow. With this method of extending of the model to handle jumps, the apparent benefits of habit destruction vanish.

Of course, one could also argue whether jumps vs. very rapid diffusions are even possible. Do habits adjust while the house is burning down? But given that a way of extending the model to continuous time that does not produce puzzling implications exists, we don't need to have that argument.

### 3 Summary and Implications

In sum, the benefits of endowment destruction, and the related puzzle that habits move in the opposite direction of current consumption, are sensitive to how one discretizes the model. If one discretizes the underlying continuous time model by the Euler approximation<sup>1</sup>, and if one examines a destruction event at the discretization frequency, we see the puzzle. If one discretizes at a high frequency, the puzzles disappear. The “right” way to discretize a continuous time model is by solving the stochastic differential equation forward for a discrete time interval to find the appropriate probability distribution  $f(s_{t+\Delta}|s_t)$ , and sample from that distribution. This discretization also does not produce the puzzle. In continuous time, one can produce the puzzle by extending the model to jumps in one particular, Euler-approximation like way. But extending the model to jumps in a way that preserves continuity between large continuous solutions and jump solutions removes the puzzle.

In our papers (Campbell and Cochrane 1999, 2000), we simulated the model using the Euler approximation at a monthly frequency, and presented implications for time-averaged annual data. The main point of our model is to show that time-varying risk aversion, induced by business cycle movements, can quantitatively explain a variety of asset pricing phenomena and their correlation with business cycles. For these results, finer or more accurate discretization made no difference. One might argue in a lawyerly way that this is the model we published, so stick with it. But one would like criticisms to be robust and not easily fixed by small changes in specification, or in this case small changes in numerical approximation procedure.

We conclude that the benefits of endowment destruction and adverse habit movements are a figment of one particular, and not very interesting, discretization and numerical approximation scheme.

While this sounds like a full-throated defense of the micro-foundations of our model, it is not. Our model, taken literally, has many predictions for the data that fail, as well as untrustworthy policy predictions. Endowment destruction is not one, but there are plenty of others. The question, which this example helps us to think about, is whether such failures undermine the main point of the model.

Taken literally, the micro-foundations of our model describe an externality in consumption. Quantities are exogenous in our model, so there is no inefficiency associated with this externality. But if one were to embed our preferences in a model with any decisions – capital, labor vs. leisure, etc. – it is likely that the consumption externality would matter and strong “policy implications” could be derived. Thus, though endowment destruction does not robustly hold in our model, strong and possibly paradoxical policy implications do hold.

Yet, as we already showed in Campbell and Cochrane (1999, p. 245), a model with internal habits, and thus no externality, would have nearly the same predictions for prices and quantities, though completely different policy and welfare conclusions. Completely different microfoundations can also produce the similar business-cycle variation in risk aversion. Models with leverage and strong bankruptcy costs also produce time-varying risk aversion; debt can function much as habit does in our model. Constantinides and Duffie (1996) can produce a time-varying business-cycle risk premium with idiosyncratic shocks, whose variance increases in bad times. Chan and Kogan (2002) and Gârleanu and Panageas (2014) produce a time-varying business-cycle risk premium

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<sup>1</sup>The “Euler approximation” of a stochastic differential equation  $dx_t = \mu(x_t)dt + \sigma(x_t)dz_t$  is the discrete-time process  $x_{t+\Delta} - x_t = \mu(x_t)\Delta + \sigma(x_t)\varepsilon_{t+\Delta}$ ;  $\varepsilon_{t+\Delta} \sim N(0, 1)$ .

with heterogeneous preferences in complete markets; the high-beta rich lose more in bad times, so the representative agent is more risk averse. Guvenen (2009) points out that a model with limited participation in risky asset markets can behave like a model with habit formation, where the consumption of nonparticipants plays the role of habit. These models all have drastically different micro-foundations and policy implications.

Most directly, in our model one can add to the utility function *any* function of aggregate consumption  $v(c_t^a)$ , thereby change the welfare implications of endowment addition or destruction completely, while not changing at all the individual's first order conditions and therefore asset pricing and quantity predictions. That policy implications have nothing to do with predictions for data is a theorem, not an observation.

Moreover, judging a model by its policy implications is a questionable test of the model's validity. If indeed habits are irreplaceable micro-foundations to time-varying business-cycle-related risk aversion, and if the right specification implies habit destruction, is that a fatal flaw? Cigarette and heroin destruction are largely thought to be beneficial. The study of rational (Becker and Murphy 1988 for example) and irrational addiction shows that temporally interdependent preferences can lead to all sorts of interesting dynamics. Several inequality pundits are already advocating steep taxation of higher incomes precisely to offset perceived externalities of conspicuous consumption by the wealthy. (Bagwell and Bernheim 1996 offer a formal model.) Ramadan, Lent, and celebrity purge diets all can be interpreted as choices of time-varying consumption paths in order to reset habits. Like these, even in Ljungqvist and Uhlig's calculations, consumption postponement is even better than endowment destruction. In sum, it's not obvious that one should throw out models when they make unusual policy prescriptions.

Micro-foundations matter, of course, and much of the literature sorting out these models focuses on whether the posited micro-foundations are present in micro data, and sufficient to generate the observed asset pricing phenomena. But the point here concerns the fit with aggregate data only.

A better criticism is that our model, taken literally, does not fit many aspects of the aggregate data. Most of all, it has one state variable and one shock. The data clearly have more shocks and state variables. Dividend yields and interest spreads do not move in lockstep; multiple variables forecast returns; and VAR systems predicting stock returns imply imperfectly correlated shocks to cash flows and expected returns (Campbell 1991, Campbell and Vuolteenaho 2004, Cochrane 1994, 2011). The single consumption growth shock in our habit model ties the two effects together in the model. Our model predicts 100%  $R^2$  regressions, as the surplus consumption ratio and the price-dividend ratio should be perfectly correlated.

But this situation is common throughout macroeconomics. The standard real business cycle model (Kydland and Prescott 1982, King, Plosser, and Rebelo 1988) also has one, technology, shock, thus predicting a stochastic singularity among time series, which one can reject at arbitrary  $p$  values in the data. The  $Q$  theory of investment predicts an easily-rejectable 100%  $R$ , in the regression of investment on  $Q$ .

But in none of these cases do we take these failings as "fatal flaws." Rather, the models tell good stories, and it is usually clear how to embellish the models to better fit the data.

In sum, the mappings from policy implications or micro-foundations, to basic mechanisms, to salient predictions, are not one-to-one. One has to explore various alternatives to understand what features are robust and essential to a model's important points, and what features are not. In particular, the mapping from normative conclusions to data is often tenuous: models very different normative conclusions can have almost exactly the same predictions about the data.

We conclude that our model’s ability to match some features of asset pricing and macroeconomic data do not imply that one should take seriously *any* of its “policy implications,” either as direction for policy or, if they seem silly, as criticisms of the model’s basic message.

But this robustness exercise is painstaking. Ljungqvist and Uhlig provide a valuable contribution in our model, highlighting that discretization intervals matter for normative questions, if not for the features of the data we addressed.

Good economic models are, or at least start as, quantitative parables, not literal descriptions of the economy. Realistically detailed “models of everything” whose predictions and policy implications can be taken literally are so complex that they hide basic mechanisms. The parable of the sower is not useful for its agricultural policy implications, nor do measurements of wheat yields on various kinds of terrain invalidate its message. Taking parables too far from the points they are trying to illustrate can lead to nonsense.

## 4 References

- Bagwell, Laurie Simon, and B. Douglas Bernheim. 1996. "Veblen Effects in a Theory of Conspicuous Consumption." *American Economic Review* 86, 349-373.
- Gary S. Becker and Kevin M. Murphy. 1988. "A Theory of Rational Addiction." *Journal of Political Economy* 96, 675-700.
- Campbell, John Y. 1991. "A Variance Decomposition for Stock Returns." *Economic Journal* 101,157-179.
- Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107, 205-251.
- Campbell, John Y., and John H. Cochrane. 2000. "Explaining the Poor Performance of Consumption-Based Asset Pricing Models." *Journal of Finance* 55, 2863-2878.
- Campbell, John Y. and Tuomo Vuolteenaho. 2004. "Bad Beta, Good Beta." *American Economic Review* 94,1249-1275.
- Chan, Yeung Lewis and Leonid Kogan. 2002. "Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices." *Journal of Political Economy* 110, 1255-1285.
- Cochrane, John H. 1994. "Permanent and Transitory Components of GNP and Stock Prices." *Quarterly Journal of Economics* CIX, 241-266.
- Cochrane, John H. 2011. "Discount Rates." *Journal of Finance* 66, 1047-1108.
- Constantinides, George M., and Darrell Duffie. 1996. "Asset Pricing with Heterogeneous Consumers." *Journal of Political Economy* 104, 219-240.
- Gârleanu, Nicolae, and Stavros Panageas. 2014. "Young, Old, Conservative, and Bold: The Implications of Heterogeneity and Finite Lives for Asset Pricing" *Journal of Political Economy*, forthcoming.
- Güvener, Fatih. 2009. "A Parsimonious Macroeconomic Model for Asset Pricing." *Econometrica* 77, 1711-1750.
- King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. "Production, Growth and Business Cycles I The Basic Neoclassical Model." *Journal of Monetary Economics* 21, 195-232.
- Kydland, Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50, 1345-70.
- Ljungqvist, Lars, and Harald Uhlig. 2014. "Comment on the Campbell-Cochrane Habit Model." *Journal of Political Economy*, this issue.