Appendix: Monetary Policy Drivers of Bond and Equity Risks

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A Model Solution

Let π_t^* denote the central bank's inflation target at time t. We solve the model in terms of the output gap x_t and inflation and nominal interest rate gaps:

$$\hat{\pi}_t = \pi_t - \pi_t^*, \quad (1)$$
$$\hat{i}_t = i_t - \pi_t^*. \quad (2)$$

Denote the vector of state variables by:

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{i}_t]'. \quad (3)$$

We can re-write the model dynamics in terms of the state variables as:

$$x_t = \rho^{x-} x_{t-1} + \rho^{x+} E_{t-} x_{t+1} - \psi \left(E_{t-} \hat{i}_t - E_{t-} \hat{\pi}_{t+1} \right) + u_t^{IS}, \quad (4)$$
$$\hat{\pi}_t = \rho^{\pi} \hat{\pi}_{t-1} + (1 - \rho^{\pi}) E_{t-} \hat{\pi}_{t+1} + \lambda x_t - \rho^{\pi} u_t^* + u_t^{PC}, \quad (5)$$
$$\hat{i}_t = \rho^i \hat{i}_{t-1} + (1 - \rho^i) \left[\gamma^x x_t + \gamma^{\pi} \hat{\pi}_t \right] + u_t^{MP}, \quad (6)$$
$$\pi_t^* - \pi_{t-1}^* = u_t^*. \quad (7)$$

Using $E_{t-} \hat{i}_t = \hat{i}_t - u_t^{MP}$, we can write the model as:

$$0 = F E_{t-} \hat{Y}_{t+1} + G \hat{Y}_t + H \hat{Y}_{t-1} + M u_t. \quad (8)$$

where

$$F = \begin{bmatrix} \rho^{x+} & \psi & 0 \\ 0 & (1 - \rho^{\pi}) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (9)$$
$$G = \begin{bmatrix} -1 & 0 & -\psi \\ \lambda & -1 & 0 \\ (1 - \rho^i)\gamma^x & (1 - \rho^i)\gamma^{\pi} & -1 \end{bmatrix}, \quad (10)$$
$$H = \begin{bmatrix} \rho^{x-} & 0 & 0 \\ 0 & \rho^{\pi} & 0 \\ 0 & 0 & \rho^i \end{bmatrix}, \quad (11)$$
$$M = \begin{bmatrix} 1 & 0 & \psi & 0 \\ 0 & 1 & 0 & -\rho^{\pi} \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (12)$$

We focus on solutions of the form:

$$\hat{Y}_t = P \hat{Y}_{t-1} + Q u_t. \quad (13)$$

Additional solutions, such as solutions depending on two lags of state variables, may exist, see e.g. Evans and McGough (2005). P has to satisfy:

$$F P^2 + G P + H = 0. \quad (14)$$

Following Uhlig (1999), we first solve for the generalized eigenvectors and eigenvalues of Ξ with respect to Δ, where:

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0_3 \end{bmatrix}, \quad (15)$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix}. \quad (16)$$

For three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda_1 z_1', z_1']'$, $[\lambda_2 z_2', z_2']'$, $[\lambda_3 z_3', z_3']'$, a solution is given by

$$P = \Omega \Lambda \Omega^{-1}, \quad (17)$$

where $\Lambda = diag(\lambda_1, \lambda_2, \lambda_3)$ and $\Omega = [z_1, z_2, z_3]$. Generalized eigenvalues are stable if their absolute value is < 1.

Let e_k denotes the row vector with a 1 in position k and zeros otherwise. Q has to satisfy

$$Qe_k' = -[FP + G]^{-1} M e_k' \quad k = 1, 2, 4 \quad (18)$$

$$Qe_3' = -G^{-1} M e_3' \quad (19)$$

Provided that G is nonsingular, $G \times Q \times e_3' = -Me_3' = -[\psi, 0, 1]'$ implies that $Q \times e_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, i.e. the monetary policy shock has no contemporaneous effect on x_t or $\hat{\pi}_t$.

As long as we focus on solutions of the form (13) and the matrix of lagged terms H is non-singular, the solution cannot contain arbitrary random variables, or 'sunspots'. If we were to allow for more complicated solution forms, where $\hat{Y}_t$ can depend on two lags of itself as well as current and lagged shocks, sunspot solutions may be possible (Evans and McGough, 2005).

To see that solutions of the form (13) do not allow for sunspots, suppose the contrary. Assume that for some vector of random variables ϵ_t uncorrelated with $\hat{Y}_{t-1}$ and u_t:

$$\hat{Y}_t = P\hat{Y}_{t-1} + Qu_t + \epsilon_t. \quad (20)$$

The expression (20) corresponds to the definition of sunspot equilibria, see e.g. Cho and Moreno (2011). Then substituting (20) into (8) gives the same conditions for P and M as before and:

$$(FP + G)\epsilon_t \equiv 0. \quad (21)$$

But from (14), $(FP+G)\times P = -H$ is non-singular. Therefore, $FP+G$ is non-singular and $\epsilon_t \equiv 0$. This completes the proof that there are no sunspot solutions.

A.1 Equilibrium Selection and Properties

We are essentially solving a quadratic matrix equation, so picking a solution amounts to picking three out of six generalized eigenvalues. We only consider dynamically stable solutions with all eigenvalues less than 1 in absolute value, yielding non-explosive solutions for the output gap, inflation gap and interest rate gap. When there are only three generalized eigenvalues with absolute values less than 1, there exists a unique dynamically stable solution. For the period 1 calibration, we have $\gamma^{\pi} < 1$ and there exist multiple real-valued, dynamically stable solutions. The period 2 and 3 calibrations have unique dynamically stable solutions.

We only consider solutions that are real-valued, and have finite entries for Q. We also require the diagonal entries of Q to be positive. This requirement means that the immediate impact of a positive IS shock on the output gap is positive rather than negative.

We apply multiple equilibrium selection criteria, which have been proposed in the literature, to rule out "bubble" or unreasonable solutions. These different equilibrium refinements are not identical, but coincide in many cases. Therefore, there exists a unique solution satisfying all criteria for a large part of our parameter space.

McCallum (1983) proposes to pick the minimum state variable solution. This solution has a minimum set of state variables and satisfies a continuity criterion. Unfortunately, Uhlig (1999) points out that implementing this criterion directly can be computationally demanding. We therefore follow Uhlig (1999) in picking the solution with the minimum absolute eigenvalues, which under certain conditions coincides with the minimum state variable solution (McCallum 2004).

We also require that our solution is locally E-stable (Evans 1985, 1986, Evans and Honkapohja 1994) as a plausible necessary, but not sufficient, condition. Local E-stability intuitively requires that the solution is learnable. If agents expectations deviate slightly from equilibrium dynamics, the system will return to an E-stable equilibrium under a simple revision rule.

Finally, we ensure uniqueness of our solution by requiring that it equals the forward solution of Cho and Moreno (2011). The forward solution is obtained by imposing a zero terminal condition. Expectations about shocks far in the future do not affect the current equilibrium. Viewed differently, if we assume that all state variables are constant from time t+T onwards, we can solve for the time t output gap, inflation gap, and interest rate gap recursively. The forward solution obtains by letting T go to infinity.

Let *vec* denote vectorization. Applying Proposition 1.3 of Fudenberg and Levine (1998, p.25) the E-stability condition translates into the requirement that the eigenvalues of the derivative

$$\frac{\partial vec([FP+G]^{-1}H)}{\partial vec(P)} \tag{22}$$

have eigenvalues with absolute values less than 1.

We implement the Cho and Moreno (2011) criterion by requiring that the following sequence $P_n, n = 0, 1, ...$ converges to P

$$P_0 = 0_{3\times 3} \tag{23}$$
$$P_{n+1} = -[FP_n + G]^{-1} \times H \tag{24}$$

This sequence P_n has at most one limit and therefore this selection criterion yields a unique solution.

A.2 Solving for SDF and Model Dynamics Simultaneously

We can solve for the matrices P and Q in terms of the model coefficients ρ^{x-}, ρ^{x+}, ψ, ρ^{π}, λ, ρ^{i}, γ^{x}, γ^{π}.

We now want to solve the model for a given slope coefficient of volatility with respect to the output gap b and the variance-covariance matrix Σ_u. We therefore solve for the slope coefficients ρ^{x-} and ρ^{x+} in terms of the preference parameters and volatilities.

We have that

$$\rho^{x-} = \frac{\theta}{1+\theta^*} \tag{25}$$
$$\rho^{x+} = \frac{1}{1+\theta^*} \tag{26}$$
$$\psi = \frac{1}{\alpha(1+\theta^*)} \tag{27}$$
$$\theta^* = \theta - \alpha b \bar{\sigma}^2/2 \tag{28}$$
$$\bar{\sigma}^2 = Q^M \Sigma_u Q^{M\prime} \tag{29}$$
$$Q^M = e_1 Q - (1+\theta^*)e_1 \tag{30}$$

θ^* is therefore a fixed point:

$$\theta^* = \theta - \frac{1}{2}\alpha b \left(e_1 Q - (1+\theta^*)e_1\right) \Sigma_u \left(e_1 Q - (1+\theta^*)e_1\right)' \tag{31}$$

This fixed point therefore depends on the matrix Q, which depends on the solution for state variable dynamics. It would therefore substantially complicate the solution if we wanted to hold b constant across sub periods.

A.3 Bond Returns

We solve for nominal and real bond log return surprises in terms of the fundamental vector of shocks u_t. We use the loglinear framework of Campbell and Ammer (1993) and do not impose the Expectations Hypothesis. We maintain our previous simplifying approximation that risk premia on one period nominal bonds equal zero. Risk premia on longer-term bonds are allowed to vary.

We write $r_{n-1,t+1}$ for the real one-period return on a real n-period bond from time t to time t+1 and $xr_{n-1,t+1}$ for the corresponding return in excess of r_t. $r^{\$}_{n-1,t+1}$ denotes the nominal one-period return on a similar nominal bond and $xr^{\$}_{n-1,t+1}$ the corresponding excess return over i_t. We use the identities:

$$r^{\$}_{n-1,t+1} - E_t r^{\$}_{n-1,t+1} = -(E_{t+1} - E_t) \sum_{j=1}^{n-1} \left(\hat{i}_{t+j} + \pi^*_{t+j} \right) \quad (32)$$

$$- (E_{t+1} - E_t) \sum_{j=1}^{n-1} xr^{\$}_{n-j-1,t+1+j} \quad (33)$$

$$r_{n-1,t+1} - E_t r_{n-1,t+1} = -(E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} \quad (34)$$

$$- (E_{t+1} - E_t) \sum_{j=1}^{n-1} xr_{n-j-1,t+1+j} \quad (35)$$

We now derive recursive expressions for unexpected nominal and real bond returns. We guess the functional forms:

$$E_t xr^{\$}_{n-1,t+1} = (1 - bx_t) b^{\$,n} \quad (36)$$

$$E_t xr_{n-1,t+1} = (1 - bx_t) b^{n} \quad (37)$$

The functional forms (36) and (37) hold for $n = 1$ with $b^{\$,1} = b^1 = 0$. Assuming (36) and (37) for maturities less than n, we can express (33) and (35) as:

$$-(E_{t+1} - E_t) \sum_{j=1}^{n-1} xr^{\$}_{n-j-1,t+1+j} = b \sum_{j=1}^{n-1} b^{\$,n-j} e_1 P^{j-1} Q u_{t+1} \quad (38)$$

$$-(E_{t+1} - E_t) \sum_{j=1}^{n-1} xr_{n-j-1,t+1+j} = b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q u_{t+1} \quad (39)$$

We can express (32) and (34) as:

$$
\begin{aligned}
-(E_{t+1}-E_t)\sum_{j=1}^{n-1}(\hat{i}_{t+j}+\pi^*_{t+j}) &= -e_3\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Qu_{t+1} \\
&\quad -(n-1)u^*_{t+1} \qquad (40)\\
-(E_{t+1}-E_t)\sum_{j=1}^{n-1}r_{t+j} &= -(e_3-e_2P)\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Qu_{t+1} \qquad (41)
\end{aligned}
$$

Denoting

$$
\begin{aligned}
S^{\$,n} &= -(n-1)e_4-e_3\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Q, \qquad (42)\\
S^{n} &= -(e_3-e_2P)\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Q, \qquad (43)
\end{aligned}
$$

we obtain:

$$
\begin{aligned}
r^{\$}_{n-1,t+1}-E_t r^{\$}_{n-1,t+1} &= \underbrace{\left[S^{\$,n}+b\sum_{j=1}^{n-1}b^{\$,n-j}e_1P^{j-1}Q\right]}_{A^{\$,n}}u_{t+1}, \qquad (44)\\
r_{n-1,t+1}-E_t r_{n-1,t+1} &= \underbrace{\left[S^{n}+b\sum_{j=1}^{n-1}b^{n-j}e_1P^{j-1}Q\right]}_{A^{n}}u_{t+1}. \qquad (45)
\end{aligned}
$$

The conditional expected return adjusted for Jensen's inequality equals the conditional covariance between bond excess returns and marginal utility. It hence follows that:

$$
\begin{aligned}
b^{\$,n} &= \alpha\left[S^{\$,n}+b\sum_{j=1}^{n-1}b^{\$,n-j}e_1P^{j-1}Q\right]\Sigma_u Q^{M\prime} \qquad (46)\\
&\quad -\frac{1}{2}\left[S^{\$,n}+b\sum_{j=1}^{n-1}b^{\$,n-j}e_1P^{j-1}Q\right]\Sigma_u\left[S^{\$,n}+b\sum_{j=1}^{n-1}b^{\$,n-j}e_1P^{j-1}Q\right]^{\prime} \qquad (47)
\end{aligned}
$$

Similarly, we obtain the recursive expression:

$$
\begin{aligned}
b^{n} &= \alpha\left[S^{n}+b\sum_{j=1}^{n-1}b^{n-j}e_1P^{j-1}Q\right]\Sigma_u Q^{M\prime} \qquad (48)\\
&\quad -\frac{1}{2}\left[S^{n}+b\sum_{j=1}^{n-1}b^{n-j}e_1P^{j-1}Q\right]\Sigma_u\left[S^{n}+b\sum_{j=1}^{n-1}b^{n-j}e_1P^{j-1}Q\right]^{\prime}. \qquad (49)
\end{aligned}
$$

Up to a constant, log yields of nominal and real zero coupon bonds then equal:

$$y^{\$}_{n,t} = \frac{1}{n}E_t\sum_{j=0}^{n-1} r^{\$}_{n-j-1,t+1+j} \tag{50}$$

$$= \frac{1}{n}E_t\sum_{j=0}^{n-1} i_{t+j} - \frac{1}{n}E_t\sum_{j=0}^{n-1} bb^{\$,n-j}x_{t+j} \tag{51}$$

$$= \pi^*_t + \underbrace{\left[\frac{1}{n}e_3[I-P]^{-1}[I-P^n] - \frac{b}{n}\sum_{j=0}^{n-1} b^{\$,n-j}e_1P^j\right]}_{\Gamma^{\$,n}}\hat{Y}_t \tag{52}$$

$$y_{n,t} = \underbrace{\left[\frac{1}{n}(e_3-e_2P)[I-P]^{-1}[I-P^n] - \frac{b}{n}\sum_{j=0}^{n-1} b^{n-j}e_1P^j\right]}_{\Gamma^{n}}\hat{Y}_t \tag{53}$$

We can then calculate the conditional slope of the term structure as follows:

$$y^{\$}_{n,t} - i_t = \frac{1}{n}E_t\sum_{j=0}^{n-1} i_{t+j} - i_t + \frac{1}{n}E_t\sum_{j=0}^{n-1} b^{\$,n-j}(1-bx_{t+j}) \tag{54}$$

$$= \frac{1}{n}E_t\sum_{j=0}^{n-1} \hat{i}_{t+j} - \hat{i}_t + \frac{1}{n}E_t\sum_{j=0}^{n-1} b^{\$,n-j}(1-bx_{t+j}) \tag{55}$$

$$= \left(\Gamma^{\$,n} - e_3\right)\hat{Y}_t + \frac{1}{n}\sum_{j=0}^{n-1} b^{\$,n-j} \tag{56}$$

With $\hat{Y}_t$ mean zero, the average slope of the term structure and the average conditional expected bond excess return are:

$$E\left(y^{\$}_{n,t} - i_t\right) = \frac{1}{n}\sum_{j=0}^{n-1} b^{\$,n-j} \tag{57}$$

$$E\left(E_t xr^{\$}_{n-1,t+1} + \frac{1}{2}Var_t(xr^{\$}_{n-1,t+1})\right) = \alpha A^{\$,n}\Sigma_u Q^{M\prime} \tag{58}$$

A.4 Stock Returns

Modeling stocks as a levered claim on the output gap x_t, we assume that dividends are given by:

$$d_t = \delta x_t. \tag{59}$$

We interpret δ as capturing a broad concept of leverage, including operational leverage.

We write r^e_{t+1} for the log stock return and xr^e_{t+1} for the log stock return in excess of r_t. Following Campbell (1991) we decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

$$r^e_{t+1} - E_t r^e_{t+1} = \delta \left(E_{t+1} - E_t\right) \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} - \left(E_{t+1} - E_t\right) \sum_{j=1}^{\infty} \rho^j r_{t+j} - \left(E_{t+1} - E_t\right) \sum_{j=1}^{\infty} \rho^j xr^e_{t+1+j} \quad (60)$$

ρ is a loglinearization constant close to 1. Now guess the functional form:

$$E_t xr^e_{t+1} = (1 - bx_t)b^e. \quad (61)$$

Then:

$$r^e_{t+1} - E_t r^e_{t+1} = \left(\kappa A^x + A^r\right) u_{t+1}, \quad (62)$$

where

$$A^x = e_1[I - \rho P]^{-1} Q, \quad (63)$$

$$A^r = -\rho(e_3 - e_2 P)[I - \rho P]^{-1} Q, \quad (64)$$

$$\kappa = \delta(1 - \rho) + \rho \times b \times b^e. \quad (65)$$

We also write:

$$A^e = \left(\kappa A^x + A^r\right). \quad (66)$$

κA^x captures the stock returns' exposure to long-term news about the output gap. A^r captures the exposure of stock returns to real interest rate news.

The conditional equity premium adjusted for Jensen's inequality equals the conditional covariance of excess stock returns and marginal utility:

$$E_t xr^e_{t+1} + \frac{1}{2} Var_t\left(xr^e_{t+1}\right) = \alpha Cov_t(r^e_{t+1}, s_{t+1} + c_{t+1}) \quad (67)$$

$$= \alpha A^e \Sigma_u Q^{M\prime}(1 - bx_t) \quad (68)$$

The average conditional equity premium is then given by:

$$E\left(E_t xr^e_{t+1} + \frac{1}{2} Var_t\left(xr^e_{t+1}\right)\right) = \alpha A^e \Sigma_u Q^{M\prime} \quad (69)$$

It then follows that expected stock returns indeed take the hypothesized form, where κ is the positive root of the quadratic equation:

$$0 = \kappa^2 + \kappa \times 2 \frac{(\rho b)^{-1} - \alpha A^x \Sigma_u Q^{M\prime} + A^x \Sigma A^{r\prime}}{A^x \Sigma_u A^{x\prime}} + \frac{-2\delta(1-\rho)(\rho b)^{-1} + A^r \Sigma_u A^{r\prime} - 2\alpha A^r \Sigma_u Q^{M\prime}}{A^x \Sigma_u A^{x\prime}} \quad (70)$$

Applying the Campbell and Shiller (1988) approximate loglinear present value model to equity prices (ignoring constants), we obtain log dividend price ratio:

$$d_t - p_t = -\delta E_t \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} + E_t \sum_{j=0}^{\infty} \rho^j (r^e_{t+1+j} - r_{t+j}) + E_t \sum_{j=0}^{\infty} \rho^j r_{t+j} \quad (71)$$

$$= [\delta e_1 (I - P) - (b \times b^e) e_1 + e_3 - e_2 P] [I - \rho P]^{-1} \hat{Y}_t. \quad (72)$$

The model has implications for the relation between the log dividend price ratio and expected long-term excess stock returns. Denoting the k-period log equity return in excess of short-term real T-bills by $xr^e_{t \to t+k}$:

$$E_t xr^e_{t \to t+k} = -(b \times b^e) e_1 [I - P]^{-1} [I - P^k] \hat{Y}_t. \quad (73)$$

A.4.1 Bond-Stock Covariances

The conditional nominal and real bond-stock return covariances equal:

$$Cov_t(r^e_{t+1}, r^{\$}_{n-1,t+1}) = A^{\$,n} \Sigma_u A^{e\prime} (1 - bx_t) \quad (74)$$

$$Cov_t(r^e_{t+1}, r_{n-1,t+1}) = A^n \Sigma_u A^{e\prime} (1 - bx_t) \quad (75)$$

The nominal bond return loadings $A^{\$,n}$, as defined in in (44), contain a term $-(n-1) \times [0, 0, 0, 1]$ increasing linearly in bond duration and for long-term nominal bonds this is the dominant term. If a positive shock to the inflation target increases stock returns, this term contributes negatively to the nominal bond-stock covariance. If a positive shock to the inflation target decreases stock returns, this term contributes positively.

The variances of equity excess returns, nominal and real bond excess returns are:

$$Var_t(r^e_{t+1}) = A^e \Sigma_u A^{e\prime} (1 - bx_t), \quad (76)$$

$$Var_t(r^{\$}_{n-1,t+1}) = A^{\$,n} \Sigma_u A^{\$,n\prime} (1 - bx_t), \quad (77)$$

$$Var_t(r_{n-1,t+1}) = A^n \Sigma_u A^{n\prime} (1 - bx_t). \quad (78)$$

The conditional stock market betas of nominal and real bonds are independent of x_t and given by:

$$\beta_t(r^{\$}_{n-1,t+1}) = \frac{A^{\$,n} \Sigma_u A^{e\prime}}{A^e \Sigma_u A^{e\prime}}, \quad (79)$$

$$\beta_t(r_{n-1,t+1}) = \frac{A^n \Sigma_u A^{e\prime}}{A^e \Sigma_u A^{e\prime}}. \quad (80)$$

A.5 Estimable VAR(1) in Output, Inflation, and Nominal Yields

While standard empirical measures are available for the output gap, we do not observe the interest rate and inflation gaps. We therefore cannot directly estimate the recursive law of motion (13). However, for a long-term bond maturity n, we can estimate a VAR(1) in the vector:

$$Y_t = \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ y^{\$}_{n,t} \end{bmatrix} \tag{81}$$

$$= A \left[\hat{Y}_t, \pi^*_t \right]'. \tag{82}$$

The model implies that:

$$Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}. \tag{83}$$

$$\tag{84}$$

Here:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ & \Gamma^{\$,n} & & 1 \end{bmatrix}, \tag{85}$$

$$P^Y = A \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} A^{-1}, \tag{86}$$

$$Q^Y = A \begin{bmatrix} & Q & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{87}$$

$$u^Y_t = u_t \tag{88}$$

provided that the inverse of A exists.

A.6 Unconditional Second Moments

Expressions (74) through (80) allow us to calculate conditional covariances, variances, and betas, conditional on the output gap being at zero. This section shows that unconditional second moments of bond and stock returns are equal to the conditional second moments, evaluated at $x_t = 0$. The law of total variance says that for any random variables X_1 and X_2:

$$Var(X_1) = E(Var(X_1|X_2)) + Var(E(X_1|X_2). \tag{89}$$

We now apply the law of total variance to the unexpected equity return $r^e_{t+1} - E_t r^e_{t+1}$ and the output gap x_t. The unconditional variance of $r^e_{t+1} - E_t r^e_{t+1}$ is given by:

$$\begin{aligned} Var(r^e_{t+1} - E_t r^e_{t+1}) &= E\left[Var\left(r^e_{t+1} - E_t r^e_{t+1}\middle| x_t\right)\right] && (90)\\ &\quad +Var\left(E\left(r^e_{t+1} - E_t r^e_{t+1}\middle| x_t\right)\right) && (91)\end{aligned}$$

But $E\left(r^e_{t+1} - E_t r^e_{t+1}\middle| x_t\right) = 0$ for any value of x_t and therefore

$$\begin{aligned} Var(r^e_{t+1} - E_t r^e_{t+1}) &= E\left[Var\left(r^e_{t+1} - E_t r^e_{t+1}\middle| x_t\right)\right] && (92)\\ &= E\left[A^e \Sigma_u A^{e\prime}(1 - bx_t)\right] && (93)\\ &= A^e \Sigma_u A^{e\prime}. && (94)\end{aligned}$$

The expression (94) shows that the unconditional variance of equity returns equals the conditional variance of equity returns, evaluated at $x_t = 0$. It similarly follows that:

$$\begin{aligned} Var(r^{\$}_{n-1,t+1}) &= A^{\$,n}\Sigma_u A^{\$,n} && (95)\\ Var(r_{n-1,t+1}) &= A^n \Sigma_u A^n && (96)\\ Cov\left(r^e_{t+1}, r^{\$}_{n-1,t+1}\right) &= A^{\$,n}\Sigma_u A^{e\prime} && (97)\\ Cov\left(r^e_{t+1}, r_{n-1,t+1}\right) &= A^n \Sigma_u A^{e\prime} && (98)\\ & && (99)\end{aligned}$$

The unconditional betas of nominal and real bonds are therefore also equal to the conditional betas (79) and (80).

It is useful to be able to simulate unconditional second moments of model real interest rates, dividend-price ratios etc. We show that we can simulate those moments by simulating a conditionally homoskedastic VAR(1) with matrix of slope coefficients P and a conditionally homoskedastic, independently and identically distributed vector of innovations.

First, we show that the unconditional second moments of the state variables $\hat{Y}_t$ are the same as those for a conditionally homoskedastic VAR(1) with matrix of slope coefficients P and conditionally homoskedastic, independently and identically distributed vector of innovations $\epsilon_t \sim N(0, Q\Sigma_u Q')$. We denote such a conditionally homoskedastic VAR(1) process by $\tilde{Y}$. The fundamental errors of $\hat{Y}_t$ are given by:

$$\hat{Y}_t - E(\hat{Y}_t | \hat{Y}_{t-1}, \hat{Y}_{t-2}, ...) = Qu_t. \quad (100)$$

The vector of fundamental errors is uncorrelated across time and it therefore is vector white noise (Chapter 10, Hamilton 1994). Applying again the law of total variance, we obtain the unconditional variance-covariance matrix of the fundamental errors:

$$\begin{aligned} Var(Qu_t) &= E\left(Var\left(Qu_t|\hat{Y}_{t-1}\right)\right) + Var\left(E\left(Qu_t|\hat{Y}_{t-1}\right)\right) && (101)\\ &= Q\Sigma_u Q'. && (102)\end{aligned}$$

We can then apply Wold's theorem for vector processes (Chapter 10, Hamilton 1994) and write $\hat{Y}_t$ as a vector $MA(\infty)$ representation:

$$\hat{Y}_t = [I - PL]^{-1} Q u_t, \tag{103}$$

where L denotes the lag operator. By Hamilton (1994) Proposition 10.2:

$$Cov(\hat{Y}_t, \hat{Y}_{t-k}) = \sum_{j=0}^{\infty} P^{j+k} Q \Sigma_u Q' P^{j\prime}. \tag{104}$$

The process $\tilde{Y}_t$ has the same variance-covariance matrix of fundamental errors as $\hat{Y}_t$. The unconditional variances and covariances of $\tilde{Y}_t$ are hence also given by (104).

Second, we can simulate unconditional second moments of the log dividend price ratio, expected stock returns, and the real short-term interest rate by first simulating $\tilde{Y}$ and then computing the short-term real interest rate, the dividend-price ratio, and expected equity excess returns according to (72), (73), and (53) with $\tilde{Y}_t$ replacing $\hat{Y}_t$. This follows from the observation that if $\hat{Y}_t$ and $\tilde{Y}_t$ have identical variances and covariances at all leads and lags, then so do any linear combinations of $\hat{Y}_t$ and $\tilde{Y}_t$. The second moments of other quantities that are linear combinations of the state variables can be simulated similarly.

B A Note on Units

Our empirical yields and returns are in annualized percent units. Log real dividends and the log output gap are in natural percent units. Our empirical units are analogous to those used by CGG. Our empirical coefficients in Table 4 in the main paper can therefore be compared directly to those in CGG.

However, the Campbell and Shiller (1988) loglinearizations, the expression for the equity premium (67) and expected bond returns (47) are expressed in natural units. We therefore solve the model in natural units and subsequently report scaled parameters and model moments reflecting our choice of empirical units. Let a superscript ^{c} denote natural units used for solving the calibrated model. Values with no superscript denote the parameters and variables corresponding to empirical units.

Our quantities in empirical units are related to quantities in calibration units according to: $x_t = 100x_t^c$, $i_t = 400i_t^c$, $\pi_t = 400\pi_t^c$, and $y_t^{\$,n} = 400y_t^{\$,n}$ and $\pi_t^* = 400\pi_t^*$. We can therefore write the model as:

$$x_t = \rho^{x-,c} x_{t-1} + \rho^{x+,c} E_{t-} x_{t+1} - \frac{\psi^c}{4} \left(E_{t-} i_t - E_{t-} \pi_{t+1} \right) + 100 \times u_t^{IS,c} \quad (105)$$
$$\pi_t = \rho^{\pi,c} \pi_{t-1} + (1 - \rho^{\pi,c}) E_{t-} \pi_{t+1} + 4\lambda^c x_t + 400 \times u_t^{PC,c} \quad (106)$$
$$i_t = \rho^{i,c} \left(i_{t-1} - \pi^*_{t-1} \right) + (1 - \rho^{i,c}) \left[4\gamma^{x,c} x_t + \gamma^{\pi,c} (\pi_t - \pi^*_t) \right] + \pi^*_t + 400 u_t^{MP,c} \quad (107)$$
$$\pi^*_t = \pi^*_{t-1} + 400 u^*_t \quad (108)$$

Equations (105) through (108) imply relations between the empirical and calibration parameters:

$$\rho^{x-} = \rho^{x-,c}, \rho^{x+} = \rho^{x+,c}, \psi = \frac{\psi^c}{4} \quad (109)$$
$$\rho^{\pi} = \rho^{\pi,c}, \lambda = 4\lambda^c \quad (110)$$
$$\rho^{i} = \rho^{i,c}, \gamma^x = 4\gamma^{x,c}, \gamma^{\pi} = \gamma^{\pi,c} \quad (111)$$
$$\bar{\sigma}^{IS} = 100\bar{\sigma}^{IS,c}, \bar{\sigma}^{PC} = 400\bar{\sigma}^{PC,c}, \bar{\sigma}^{MP} = 400\bar{\sigma}^{MP,c}, \bar{\sigma}^* = 400\bar{\sigma}^* \quad (112)$$

Fuhrer (1997) estimates a Phillips curve with both backward-looking and forward-looking components. Using inflation in annualized percent, and the log output gap in natural units, he find a backward-looking component of 0.8, a forward-looking component of 0.2, and a weight on the output gap of 0.12. We can therefore compare the parameter λ in empirical units directly to the magnitudes in CGG, Fuhrer (1997), and Roberts (1995).

Yogo (2004) scales interest rates and inflation to quarterly units. Our calibrated values for ψ^c in natural units can therefore be compared directly to the estimated values in Yogo (2004). We therefore report the value ψ^c corresponding to natural units rather than ψ corresponding to empirical units throughout the paper.

We choose the leverage parameter δ to match the relative volatilities of log real dividend growth and log output gap growth. We use four quarter growth rates to smooth out some of the more seasonal fluctuations. We consider four quarter log output growth $\Delta x_t = x_t - x_{t-4}$. The standard deviation of this growth rate over the period 1960.Q1-2011.Q4 is 2.20%. Let d_t denote the sum of log S&P 500 real dividends. Monthly real S&P 500 dividends are from Robert Shiller's web site. These real dividends are deflated by the not seasonally adjusted CPI-U with a basis of 1982-84=100. We obtain quarterly dividends by summing the level real dividends within the quarter. The standard deviation 1960.Q1-2011.Q4 of the four quarter log dividend growth rate $\Delta d_t = d_t - d_{t-4}$ equals 5.35%. Our model specifies dividends as a levered claim on the output gap with $d_t = \delta x_t$. We therefore set the leverage parameter δ to match the relative standard deviations of output and dividend growth. This gives $\delta = 2.43$.

Due to our choice of empirical units, we use a slightly different transformation from the transition matrix P^c of the state variables in natural unit to the transition matrix P^Y of the estimable VAR(1). We have the relation:

$$
\begin{aligned}
Y_t &= \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ y^{\$}_{n,t} \end{bmatrix} && (113)\\
&= A^c \left[\hat{x}^c_t, \hat{\pi}^c_t, \hat{i}^c_t, \pi^{c,*}_t\right]', && (114)
\end{aligned}
$$

where:

$$
\begin{aligned}
A^c &= diag(100, 400, 400, 400) \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ & \Gamma^{\$,n} & & 1 \end{bmatrix}, && (115)\\
P^Y &= A^c \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} A^{c-1}, && (116)\\
Q^Y &= A^c \begin{bmatrix} & Q & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, && (117)\\
u^{c,Y}_t &= \left[u^{c,IS}_t,\ u^{c,PC}_t,\ u^{c,MP}_t,\ u^{c,*}_t\right]', && (118)\\
Y_t &= P^Y Y_{t-1} + Q^Y u^{c,Y}_{t+1}. && (119)
\end{aligned}
$$

We report annualized percent standard deviations of equity and bond returns at the average output gap $x_t = 0$. We calculate annualized standard deviations of equity and bond returns in percent at $x_t = 0$:

$$
\begin{aligned}
Std_t(r^e_{t+1}) &= 200\sqrt{A^e \Sigma_u A^{e\prime}}, && (120)\\
Std_t(r^{\$}_{n-1,t+1}) &= 200\sqrt{A^{\$,n} \Sigma_u A^{\$,n\prime}}, && (121)\\
Std_t(r_{n-1,t+1}) &= 200\sqrt{A^n \Sigma_u A^{n\prime}}. && (122)
\end{aligned}
$$

We back out empirical shocks for each sub period separately. From the empirical series $Y^{emp}_t = \left[x^{emp}_t, \pi^{emp}_t, i^{emp}_t, y^{emp,\$,n}_t\right]$, we back out fundamental shocks in empirical units:

$$
\begin{aligned}
\left[u^{IS}_t, u^{PC}_t, u^{MP}_t, u^*_t\right]' &= \left[100u^{c,IS}_t,\ 400u^{c,PC}_t,\ 400u^{c,MP}_t,\ 400u^{c,*}_t\right]', && (123)\\
&= diag(100, 400, 400, 400) \times Q^{Y-1}\left(Y^{emp}_t - P^Y Y^{emp}_{t-1}\right). && (124)
\end{aligned}
$$

We transform the parameter b into empirical units according to $b = b^c/100$. Then $(1 - b^c x^c_t) = (1 - b x_t)$. We calculate the standard deviation of the volatility factor $(1 - b x_t)$ at $x_t = 0$ according to $b\sqrt{e_1 Q \Sigma_u Q' e_1'}$.

B.1 Partial Derivatives

We compute the partial derivative of the nominal bond beta with respect to $ln(\bar{\sigma}_u^k)$ as follows:

$$\frac{\partial \beta^{\$}}{\partial ln \bar{\sigma}_u^k}| = \frac{1}{A^e \Sigma_u A^{e\prime}} 2A^{\$,n} e_k^{\prime} e_k A^{e\prime} \bar{\sigma}_u^{k2} \quad (125)$$

$$\frac{\partial Std_t(r_{t+1}^e)}{\partial ln \bar{\sigma}_u^k} = \frac{200 A^e e_k^{\prime} e_k A^{e\prime} \bar{\sigma}_u^{k2}}{\sqrt{A^e \Sigma_u A^{e\prime}}} \quad (126)$$

$$\frac{\partial Std(r_{n-1,t+1}^{\$})}{\partial ln \bar{\sigma}_u^k} = \frac{200 A^{\$,n} e_k^{\prime} e_k A^{\$,n\prime} \bar{\sigma}_u^{k2}}{\sqrt{A^{\$,n} \Sigma_u A^{\$,n\prime}}} \quad (127)$$

The partial derivatives for the nominal bond beta sum to two times the calibrated nominal bond beta for each sub period. The partial derivatives for the standard deviations of asset returns sum to the calibrated standard deviation of asset returns for each sub period.

C Details of Moment Fitting Procedure

We minimize the distance between model and empirical moments summed over all three sub-periods. We use a superscript p to denote period p moments and a hat to denote empirically estimated moments. Our objective function is:

$$Obj = \sum_{p=1}^{3} \left[\left\| P^{Y,p} - \hat{P}^{Y,p} \right\|^2 + \left\| diag(Q^{Y,p} \Sigma_u Q^{Y,p}) - diag(\widehat{Q^{Y,p} \Sigma_u Q}^{Y,p}) \right\|^2 \right. \quad (128)$$

$$+ \left(\frac{1}{10} (Std^p(r_{n-1,t+1}^{\$}) - \widehat{Std^p(r_{n-1,t+1}^{\$})})\right)^2 \quad (129)$$

$$+ \left(\frac{1}{10} (Std^p(r_{t+1}^e) - \widehat{Std^p(r_{nt+1}^e)})\right)^2 \quad (130)$$

$$\left. + \left(10 \times (\beta^p(r_{n-1,t+1}^{\$}) - \widehat{\beta^p(r_{n-1,t+1}^{\$})})\right)^2 \right] \quad (131)$$

We optimize Obj over the following parameters: ρ^{x-}, ρ^{x+}, $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$, $p = 1, 2, 3$. We hold all other parameters constant at the values shown in Table 5 in the main paper.

In order to reduce the dimensionality of the minimization problem, we minimize Obj iteratively. First, we minimize with respect to the standard deviations of shocks while holding the Euler equation parameters ρ^{x+} and ρ^{x-} constant at initial guesses. Second, we minimize with respect to ρ^{x+} and ρ^{x-} while holding constant the standard deviations of shocks at their optimal values from the first step. Third, we minimize again with respect to the standard deviations of shocks holding constant ρ^{x+} and ρ^{x-} at their optimal values from the second step.

Step 1: Starting from an initial guess of $\rho^{x-} = 0.4503$ and $\rho^{x+} = 0.6161$, we first minimize with respect to $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$ holding ρ^{x-} and ρ^{x+} constant. Given ρ^{x-} and ρ^{x+}, we can minimize with respect to $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$ independently for each period p.

We use a simple and robust minimization procedure. We randomly draw 50000 parameter vectors. We draw $\bar{\sigma}^{IS,p}, \bar{\sigma}^{PC,p}, \bar{\sigma}^{MP,p}, \bar{\sigma}^{*,p}$ from independent uniform distributions. Our support intervals for 1960.Q1-1979.Q2 are such that $\left[(\bar{\sigma}^{IS,1}), (\bar{\sigma}^{PC,1}), (\bar{\sigma}^{MP,1}), (\bar{\sigma}^{*,1})\right] \in [0,0,0,0] \times [0.8256, 2.2069, 2.2768, 0.7715]$. Our support intervals for 1979.Q3-1996.Q4 are such that $\left[(\bar{\sigma}^{IS,2}), (\bar{\sigma}^{PC,2}), (\bar{\sigma}^{MP,2}), (\bar{\sigma}^{*,2})\right] \in [0,0,0,0] \times [0.7800, 1.3338, 4.3310, 1.1228]$. Our support intervals for 1997.Q1-2011.Q4 are such that $\left[(\bar{\sigma}^{IS,3}), (\bar{\sigma}^{PC,3}), (\bar{\sigma}^{MP,3}), (\bar{\sigma}^{*,3})\right] \in [0,0,0,0] \times [0.6153, 1.8542, 0.8119, 1.0595]$. Minimizing with respect to the volatilities of shocks for each sub sample yields:

$$\begin{pmatrix} \bar{\sigma}^{IS,1} & \bar{\sigma}^{IS,2} & \bar{\sigma}^{IS,3} \\ \bar{\sigma}^{PC,1} & \bar{\sigma}^{PC,2} & \bar{\sigma}^{PC,3} \\ \bar{\sigma}^{MP,1} & \bar{\sigma}^{MP,2} & \bar{\sigma}^{MP,3} \\ \bar{\sigma}^{*,1} & \bar{\sigma}^{*,2} & \bar{\sigma}^{*,3} \end{pmatrix} = \begin{pmatrix} 0.38 & 0.54 & 0.34 \\ 1.09 & 0.83 & 0.90 \\ 1.23 & 1.93 & 0.38 \\ 0.37 & 0.72 & 0.51 \end{pmatrix} \tag{132}$$

Step 2: In the second step, we minimize with respect to ρ^{x+} and ρ^{x-} while holding the volatilities of shocks constant at the values shown in (132). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0,1]$ and $U_2 \in [0,1]$ and set $\rho^{x-} = 0.4253 + 0.05U_1$ and $\rho^{x+} = (1 - \rho^{x-}) + 0.2 \times \rho^{x-}U_2$, thereby ensuring that ρ^{x+} and ρ^{x-} sum to more than 1. We obtain minimizing parameter values $\rho^{x-} = 0.4466$ and $\rho^{x+} = 0.6224$ agreeing with the initial guesses up to two significant digits. Figure A.1 shows the objective function Obj against U_1 and U_2. Each dot corresponds to one combination of parameter values. Figure A.1 shows that the optimizing parameter values are in the middle of the ranges considered. We therefore are not at a boundary solution. Moreover, the optimal parameter values occur at a clear minimum, indicating that the parameters ρ^{x+} and ρ^{x-} are well identified.

Step 3: The third step is exactly the same as the first step, except that we hold ρ^{x-} and ρ^{x+} constant at their new values. Figure A.2 shows the objective function against the standard deviations of shocks for each sub sample period. If the volatilities of shocks are well identified, the lower envelopes of the scatter plots in Figure A.2 should have clear minima. It appears that the objective function exhibits clear minima with respect to each of the shock volatilities. Figure A.2 shows that all volatilities are in the interior of the intervals that we are optimizing over. This finding is reassuring in that it suggests that we are considering sufficiently large ranges.

The optimal volatilities of shocks are shown in Table 5 in the main paper. These optimal volatilities are close to the preliminary values (132). Moreover, the final values for ρ^{x+} and ρ^{x-} are very close to the initial guesses, indicating convergence of our algorithm.

D Additional Calibration Features and Robustness

Table A.1 shows the matrix of slope coefficients for the quarterly VAR(1) in the log output gap, inflation, the Federal Funds rate, and the 5 year nominal yield both in the model and in the data. Table A.1 shows that the calibrated model can generate substantial persistence in the output gap, inflation, Fed Funds rate, and the long-term nominal yield, even though the output gap is somewhat less persistent than in the data. The off-diagonal elements are generally small and often close to zero.

Figure A.3 shows a time series of smoothed shocks backed out from our sub period calibrations. For each sub period, we back out the fundamental model shocks by inverting the relation $Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}$ and plugging in the empirical time series for the vector Y_t and the model implied matrices P^Y and Q^Y.

Tables A.2 and A.3 present an alternative calibration and are analogous to Tables 5 and 6 in the main paper. The alternative calibration fits the volatility of VAR(1) residual volatilities, and the volatilities of bond and stock returns, but not the nominal bond beta. Table A.2 shows that in the alternative calibration we obtain a lower volatility of PC shocks in the middle sub period. Consequently, the alternative calibration obtains a negative nominal bond beta in the second sub period instead of a positive nominal bond beta.

Table A.4 shows additional moments from the calibration. The equity premium is close to 4% on an annualized basis.

The model assumes that shocks are uncorrelated for parsimony. We would therefore expect that model-implied shocks should be uncorrelated. Table A.5 reports the univariate correlations between IS, PC, MP, and inflation target shocks for each subperiod calibration. The average correlations are generally close to zero. However, the correlation between the inflation target shock and monetary policy shock stands out and is negative in all three subperiods. This implied negative correlation can quite plausibly be a result of assuming a monetary policy rule, which smoothes the interest rate gap, rather than the interest rate. We therefore consider robustness to an alternative monetary policy rule in the next section.

D.1 Robustness to Alternative MP Rule

In our main formulation, the central bank smoothes the difference between the Fed Funds rate and the inflation target, following an interest rate rule of the form:

$$i_t = \rho^i (i_{t-1} - \pi^*_{t-1}) + (1-\rho^i)\left[\gamma^x x_t + \gamma^\pi (\pi_t - \pi^*_t)\right] + \pi^*_t + u^{MP}_t. \qquad (133)$$

In this section, we consider a similar, alternative formulation, which instead smoothes the level of the Fed Funds rate:

$$\begin{aligned} i_t &= \rho^i i_{t-1} + (1-\rho^i)\left[\gamma^x x_t + \gamma^\pi(\pi_t - \pi_t^*) + \pi_t^*\right] + \tilde{u}_t^{MP} & (134) \\ &= \rho^i(i_{t-1} - \pi_{t-1}^*) + (1-\rho^i)\left[\gamma^x x_t + \gamma^\pi(\pi_t - \pi_t^*)\right] + \pi_t^* - \rho^i \tilde{u}_t^* + \tilde{u}_t^{MP}. & (135) \end{aligned}$$

Expression (135) shows that the two monetary policy rules are equivalent if $u_t^{MP} = -\rho^i \tilde{u}_t^* + \tilde{u}_t^{MP}$, so the monetary policy shock in (133) will be more negatively correlated with the inflation target shock u_t^* than the monetary policy shock in (134). Considering a model calibration with $\tilde{u}_t^*$ and $\tilde{u}_t^{MP}$ independent, the alternative monetary policy rule can therefore address the negative correlation between implied MP and inflation target shocks in Table A.5.

The model solution takes exactly the same form as before, the only difference being that now:

$$M = \begin{bmatrix} 1 & 0 & \psi & 0 \\ 0 & 1 & 0 & -\rho^\pi \\ 0 & 0 & 1 & -\rho^i \end{bmatrix}. \quad (136)$$

We re-create Figures 3 and 4 with the monetary policy rule (135) using the parameter values shown in Table 5. Since we do not re-calibrate the model to match macroeconomic and asset pricing moments, we do not expect the modified model to match the empirical betas.

Figures A.4 and A.5 look very similar to Figures 3 and 4 in the main text, indicating that the identified monetary policy drivers act similarly and similarly strongly in the alternative model. Figures A.3 and A.4 differ from Figures 3 and 4 in that the read and blue regions have shifted slightly relative to the dots indicating the estimated monetary policy coefficients for each regime.

Importantly, the nominal bond beta is still strongly increasing in the inflation weight γ^π, weakly increasing in the output weight γ^x, and increasing and nonlinear in the monetary policy persistence coefficient ρ^i. Panel B of Figure A.5 indicates that even in this alternative, not fully calibrated model, the increase in monetary policy persistence would have switched the nominal bond beta from positive to negative if all other parameters had remained constant.

D.2 Robustness to Phillips Curve Parameter ρ^π

In our calibration, we use a Phillips curve with a backward looking component of $\rho^\pi = 0.8$, which is consistent with the empirical estimates of Fuhrer (1997). However, a wide range of empirical estimates are available for ρ^π are available in the literature. For instance, Gali and Gertler (1999) use the labor share of income instead of the output gap and estimate a smaller backward-looking component in the Phillips curve.

Choosing a high value of ρ^π ensures that our model has a unique solution for a large range of monetary policy functions.

We verify that the main mechanism identified in this paper is robust to choosing a lower value of ρ^π. Unfortunately, the range of parameter values with a stable and unique model solution shrinks rapidly when we choose smaller values of ρ^π. We therefore consider an alternative value for ρ^π of 0.7. Considering even smaller values would leave us with no solution for a large range of monetary policy functions.

Figures A.6 and A.7 are analogous to Figures 3 and 4 in the main paper, but they use a lower value for ρ^π. All other parameters are as in Table 5 in the main paper. We do not re-calibrate the model and we therefore do not expect the model to match empirical asset pricing moments exactly. The main difference between Figures A.6 and A.7 and Figures 3 and 4 in the main text is that in Figures A.6 and A.7 the regions where no solution exists (shown in white) are much larger. Whenever a solution exists, it is very similar to the one in the main calibration. Importantly, the dependence of the nominal bond beta on the monetary policy parameters γ^π, γ^x, and ρ^i looks very similar.

D.3 Effect of Phillips Curve Slope

There is some empirical evidence that in addition to the shock volatilities and the monetary policy rule parameters (which are allowed to vary across our subperiod calibrations), the slope of the Phillips curve. λ may also have changed over time. Smets and Wouters (2007) report some evidence that the slope of the Phillips curve may have decreased over time. We investigate whether this might explain the sign switch in the empirical nominal bond beta in the late 1990s.

Figure A.8 plots the nominal bond beta as a function of λ for all three subperiod calibrations. Figure A.8 shows that the nominal bond beta decreases in the slope of the Phillips curve λ if all other parameters are held constant at their period 2 or period 3 values. If the slope of the Phillips curve has decreased over time, our model indicates that this should have led to an increase in the nominal bond beta and not to the decrease observed in the late 1990s.

D.4 Robustness to Different Leverage Parameter

Our baseline model assumes that the output gap and dividends are perfectly correlated, while in the data the correlation is much lower. We could follow a similar route as in Campbell and Cochrane (1999) and address this issue by modeling dividends as proportional to the output gap plus an idiosyncratic shock. In order to maintain a realistic ratio of the dividend growth volatility and output gap growth volatility, we would need to reduce the sensitivity of dividends to the output gap.

Campbell and Cochrane (1999) remark that their habit formation model has very similar implications no matter whether they model dividend growth as perfectly or imperfectly correlated with consumption growth.

We study the sensitivity of our model implications with respect to the leverage parameter δ in order to understand how important the assumption of perfectly correlated dividends and output gap is for our findings. Adding an idiosyncratic, unpriced, shock to dividends is unlikely to change substantially any asset pricing dynamics. Modeling dividends as imperfectly correlated with the output gap should therefore yield implications that are very similar to reducing the leverage parameter δ.

Figure A.9 is equivalent to Figure 4 in the main text, but it sets $\delta = 1$, corresponding to an extremely low firm leverage ratio of 0%. We can see that the implications for nominal bond betas are qualitatively and quantitatively extremely similar to the baseline calibration. In fact, Figure A.9 and Figure 4 in the main paper are visually indistinguishable (except for the fact that Figure A.9 uses fewer pixels). We interpret Figure A.9 as indicating that our results are not sensitive to assuming a strong correlation between dividends and the output gap.

Figure A.10 explores the real bond beta as a function of monetary policy parameters. The figure is analogous to Figure 4, Panel A in the main paper, except that this figure plots the beta of real bonds instead of nominal bonds. We can see that the real bond beta is negative for a wide region around the period 1 monetary policy parameters, which are indicated with a diamond. Moreover, the real bond beta decreases sharply in the MP coefficient γ^{π}. Table 10 in the main paper implies that, had TIPS existed, they would have been extremely safe with a beta of -215.55. Given that the region around period 1 monetary policy parameters includes much more reasonable values for the real bond beta, we believe we could recalibrate our model relatively easily with a restriction that the implied volatility or beta of TIPS must fall within a certain range. Given that the model mechanism generally seems robust to small changes in parameters, we don't expect that this would change the model implications.

E Additional Empirical Results

Table A.6 reports Taylor rule estimates for superiod 3 (1997.Q-2011.Q4) and splits it into a pre-Lehman subsample (1997.Q1-2008.Q2) and a post-Lehman subsample (2008.Q3-2011.Q4). Interestingly, the estimates for the pre-Lehman subsample are virtually identical to the estimates for the full subperiod 3. The estimates reported in Table 4 in the main paper therefore do not appear to be mainly driven by the financial crisis, just as the negative nominal bond beta does not appear mainly driven by the financial crisis. For the post-Lehman subsample, the estimates for all monetary policy coefficients (γ^x, γ^{π}, and ρ^i) are all close to zero with small standard errors.

This is intuitive, since the most salient feature of monetary policy during the crisis was perhaps that the Federal Funds rate has been stuck at the zero lower bound. The empirical results in Table A.6 suggest that we can model post-Lehman monetary policy by setting all monetary policy parameters to zero.

Table A.7 tests for statistical significance of the changes in monetary policy parameters. It shows that the major changes (increase in γ^{π} from period 1 to period 2; increase in ρ^{i} from period 2 to period 3) are indeed statistically significant.

If changes in bond risks are driven by macroeconomic factors, then changes in bond risks should be reflected in changing macroeconomic correlations. Lower than expected inflation raises nominal bond prices, all else equal, so the inflation-output correlation should typically take the opposite sign from the bond-stock correlation.

Table A.8 compares sub-sample correlations of asset prices and macroeconomic variables. The empirical output gap is highly persistent and it is therefore unsurprising that three year equity excess returns are more strongly correlated with the output gap than highly volatile quarterly stock returns. We therefore use quarterly overlapping three year bond and stock excess returns for our comparison of asset return correlations and macroeconomic correlations. Table A.8 confirms our intuition that bond excess returns should at least partly reflect news about inflation and that equity excess returns should reflect the business cycle. In each sub period, empirical bond excess returns are negatively correlation with inflation and equity excess returns are positively correlated with the output gap.

Table A.8 confirms that the changes in the bond-stock comovement documented in Figure 1 and in Table 6 are robust to using three year returns instead of daily or quarterly returns. The correlation between three year stock returns and three year bond returns was positive and significant in the first sub-period, increased in the second sub period, and became negative and significant in the last sub period.

The bond-output gap, inflation-stock, and inflation-output gap correlations confirm our intuition that changing bond risks are related to the prevalence of inflationary recessions versus deflationary recessions during different regimes. The bond-output gap correlation typically has the same sign as the bond-stock correlation, while the inflation-stock return correlation and the inflation-output correlation has the opposite sign. The only exception to this pattern is the first sub period bond-output gap correlation, which takes a negative, but small and insignificant, value.

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Tables and Figures

Table A.1: Empirical and Model VAR(1) Matrices for Sub Periods

	1960.Q1-1979.Q2 Model				1960.Q1-1979.Q2 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.74	-0.28	-0.05	0.33	0.97	-0.05	-0.20	0.19
Inflation	0.32	0.84	-0.03	0.19	-0.04	0.42	0.37	0.42
Fed Funds Rate	0.23	0.17	0.53	0.30	0.17	0.04	0.54	0.47
Log Nom. Yield	0.07	-0.02	-0.06	1.08	0.03	0.08	0.02	0.83
	1979.Q3-1996.Q4 Model				1979.Q3-1996.Q4 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.73	-0.79	-0.08	0.87	0.90	-0.08	0.02	-0.07
Inflation	0.32	0.54	-0.05	0.51	0.03	0.78	0.10	-0.04
Fed Funds Rate	0.23	1.03	0.47	-0.50	0.07	0.76	0.11	0.65
Log Nom. Yield	0.22	-0.20	-0.07	1.27	-0.05	0.22	-0.02	0.83
	1997.Q1-2011.Q4 Model				1997.Q1-2011.Q4 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.47	-0.22	-0.16	0.39	0.93	0.04	0.03	0.13
Inflation	0.17	0.87	-0.09	0.22	0.20	0.37	-0.17	-0.06
Fed Funds Rate	0.14	0.16	0.90	-0.06	0.00	0.14	0.68	0.47
Log Nom. Yield	-0.32	-0.10	-0.13	1.23	0.07	-0.06	0.06	0.75

P^Y is the matrix of slope coefficients of a quarterly VAR(1) in the log output gap, inflation, Fed Funds rate, and 5 Year Nominal Yield.

Table A.2: Alternative Calibration Parameter Choices

Time-Invariant Parameters				
Log-Linearization Constant	ρ		0.99	
Leverage	δ		2.43	
Preference Parameter	α		30	
Backward-Looking Comp. PC	ρ^{π}		0.80	
Slope PC	λ		0.30	
Forward-Looking Comp. IS	ρ^{x+}		0.62	
Backward-Looking Comp. IS	ρ^{x-}		0.45	
Monetary Policy Rule		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^{x}	0.42	-0.07	0.44
MP Coefficient Infl.	γ^{π}	0.69	1.44	1.92
Backward-Looking Comp. MP	ρ^{i}	0.56	0.43	0.89
Std. Shocks				
Std. IS	$\bar{\sigma}^{IS}$	0.38	0.39	0.32
Std. PC shock	$\bar{\sigma}^{PC}$	1.02	0.73	0.98
Std. MP shock	$\bar{\sigma}^{MP}$	1.21	1.93	0.47
Std. infl. target shock	$\bar{\sigma}^{*}$	0.33	0.68	0.54

The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks. The time-invariant parameters and monetary policy rule parameters are identical to those in Table 5 in the main paper. The standard deviations of shocks differ from the calibration in Table 5 in the main paper.

Table A.3: Alternative Calibration Model and Empirical Moments

	60.Q1-79.Q2		79.Q3-96.Q4		97.Q1-11.Q4	
Std. VAR(1) Residuals	Empirical	Model	Empirical	Model	Empirical	Model
Output Gap	0.92	0.70	0.75	0.69	0.65	0.62
Inflation	1.12	1.14	0.89	0.84	0.80	1.08
Fed Funds Rate	1.22	1.28	2.07	2.05	0.66	0.68
Log Nominal Yield	0.48	0.42	0.85	0.77	0.55	0.60
Std. Asset Returns						
Std. Eq. Ret.	17.62	17.89	15.34	16.49	20.08	19.01
Std. Nom. Bond Ret.	4.85	3.98	9.11	6.55	5.55	5.42
Nominal Bond Beta	0.06	0.12	0.20	-0.05	-0.17	-0.04
Taylor Rule: Fed Funds onto Output, Infl. and Lag. Fed Funds						
Output	0.18	0.22	-0.04	-0.13	0.05	0.02
Inflation	0.30	0.34	0.83	0.65	0.21	0.19
Lagged Fed Funds	0.56	0.59	0.43	0.39	0.89	0.81

We compare model and empirical moments for the alternative calibration. Alternative calibration parameters are specified in Table A.2. The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks.

Table A.4: Appendix Model Moments

	Model				Empirical			
	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	Avg.	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-11.Q4
Equity Premium.	3.61	3.12	3.29	3.35	3.23	8.12	4.94	5.36
Nom. Bond Exc. Ret.	0.25	0.68	-0.50	0.18	0.01	2.31	2.97	1.64
$E\left(y^{\$}_{5,t} - i_t\right)$	0.14	0.39	-0.03	0.18	0.74	1.32	1.14	1.05
$Corr(x_t, y^{\$}_{5,t} - i_t)$	0.05	0.31	0.93	0.39	-0.62	-0.21	-0.53	-0.46
$Corr(x_t, y^{\$}_{5,t})$	0.01	-0.02	0.31	0.09	-0.08	-0.34	0.82	-0.07
$Corr(x_t, i_t)$	-0.02	-0.16	-0.20	-0.12	0.22	-0.21	0.80	0.05
$xr^{\$}_{5,t+1}$ onto $(y^{\$}_{5,t} - i_t)$	-0.08	-0.71	4.12	0.92	2.24	3.09	2.37	2.84*
$xr^{\$}_{5,t+1}$ onto x_t	-1.23	-3.56	5.27	-0.14	-0.72*	-0.17	-0.16	-0.47
$xr^{\$}_{5,t+1}$ onto dp_t	0.06	0.14	-0.30	-0.02	0.06	0.02	-0.03	-0.01

The equity premium and the average nominal bond excess return show average returns in excess of a short-term bond adjusted for Jensen's inequality. The last three rows show regression coefficients of log 5 year bond excess returns (Annualized, %) onto the slope of the yield curve (Annualized, %), the output gap (%), and the log dividend price ratio (%), respectively. The last three rows show $*$ when the coefficient is significant at the 5% level with Newey-West standard errors with two lags.

Table A.5: Correlations of Implied Shocks

1960.Q1-1979.Q2

	IS	PC	MP	Infl. Target
IS	1.00	-0.24	0.11	-0.14
PC		1.00	0.04	-0.34
MP			1.00	-0.40
Infl. Target				1.00

1979.Q3-1996.Q4

	IS	PC	MP	Infl. Target
IS	1.00	0.05	-0.58	-0.07
PC		1.00	0.07	-0.49
MP			1.00	-0.31
Infl. Target				1.00

1997.Q1-2011.Q4

	IS	PC	MP	Infl. Target
IS	1.00	-0.68	0.00	-0.24
PC		1.00	0.07	0.11
MP			1.00	-0.75
Infl. Target				1.00

For each sub period, we back out the fundamental model shocks by inverting the relation $Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}$ and plugging in the empirical time series for the vector Y_t and the model implied matrices P^Y and Q^Y.

Table A.6: Empirical Monetary Policy Function Crisis Sample

Fed Funds i_t	97.Q1-11.Q4	97.Q1-08.Q2	08.Q3-11.Q4
Output Gap	0.05	0.17	-0.02
	(0.04)	(0.12)	(0.04)
Inflation	0.21**	0.27**	-0.01
	(0.07)	(0.09)	(0.02)
Lagged Fed Funds	0.89**	0.88**	0.03
	(0.06)	(0.09)	(0.04)
Constant	-0.12	-0.27	-0.04
	(0.29)	(0.27)	(0.33)
R^2	0.91	0.86	0.22
Implied $\hat{\gamma}^x$	0.44	1.43	-0.02
	(0.21)	(0.40)	(0.05)
Implied $\hat{\gamma}^\pi$	1.92*	2.29*	-0.01
	(1.26)	(1.55)	(0.02)
Implied $\hat{\rho}^i$	0.89**	0.88**	0.03
	(0.06)	(0.09)	(0.04)

This table estimates the monetary policy rule before and after the Lehman brothers bankruptcy in 2008.Q3. All variables and test specifications are described in Table 4 in the main text.

Table A.7: Estimating Changes in the Monetary Policy Rule

Fed Funds i_t	60.Q1-96.Q4	79.Q3-11.Q4	97.Q1-11.Q4
Dummy Period $\mathcal{T}$	79.Q3-96.Q4	97.Q1-11.Q4	08.Q3-11.Q4
Output Gap x_t	0.18**	-0.04	0.17
	(0.06)	(0.13)	(0.13)
Inflation π_t	0.30**	0.83**	0.27**
	(0.07)	(0.21)	(0.09)
Lagged Fed Funds i_{t-1}	0.56**	0.43*	0.88**
	(0.10)	(0.17)	(0.09)
Output Gap×Dummy $x_t I_{t\in\mathcal{T}}$	-0.22	0.09	-0.19
	(0.14)	(0.14)	(0.13)
Inflation×Dummy $\pi_t I_{t\in\mathcal{T}}$	0.53*	-0.62**	-0.28**
	(0.22)	(0.22)	(0.09)
Lagged Fed Funds×Dummy $i_{t-1} I_{t\in\mathcal{T}}$	-0.14	0.47*	-0.85**
	(0.20)	(0.18)	(0.10)
Dummy $I_{t\in\mathcal{T}}$	0.84	-1.87	0.23
	(1.00)	(0.96)	(0.40)
Constant	0.91*	1.75	-0.27
	(0.38)	(0.92)	(0.27)
R^2	0.77	0.85	0.93
Implied $\Delta\hat{\gamma}^x$	-0.49	0.52	-1.45
	(0.25)	(0.30)	(0.41)
Implied $\Delta\hat{\gamma}^\pi$	0.75*	0.47	-2.30
	(0.25)	(1.28)	(1.59)
Implied $\Delta\hat{\rho}^i$	-0.14	0.47*	-0.85
	(0.20)	(0.18)	(0.10)

Variables and tests are described in Table 4 in the main text. We estimate $i_t = c^0 + c^x x_t + c^\pi \pi_t + c^i i_{t-1} + d^0 I_{t\in\mathcal{T}} + d^x x_t I_{t\in\mathcal{T}} + d^\pi \pi_t I_{t\in\mathcal{T}} + d^0 i_{t-1} I_{t\in\mathcal{T}} + \epsilon_t$. Changes in monetary policy parameters, such as $\Delta\hat{\gamma}^x$, show estimated changes from the pre-$\mathcal{T}$ sub-sample to the $\mathcal{T}$ sub-sample. Standard errors for $\Delta\hat{\gamma}^x$ and $\Delta\hat{\gamma}^\pi$ are calculated by the delta method. Significance levels for changes in monetary policy parameters are based on a likelihood ratio test.

Table A.8: Sub-Period Correlations of Bond Returns, Stock Returns, Output Gap, and Inflation

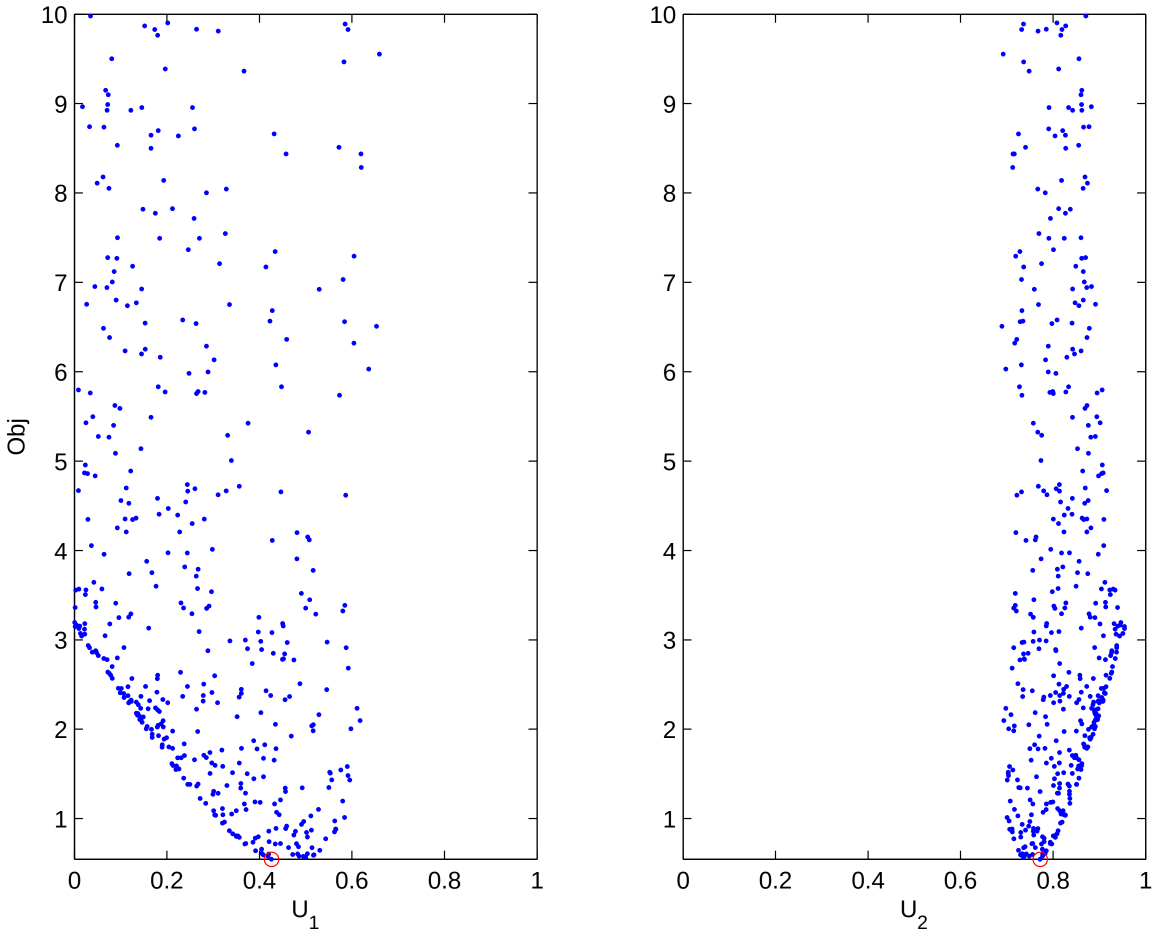
60.Q1-79.Q2	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.32*	1		
Output Gap	-0.17	0.36*	1	
Inflation	-0.25*	-0.60*	-0.14	1

79.Q3-96.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.46*	1		
Output Gap	0.30*	0.23	1	
Inflation	-0.74*	-0.27*	-0.16	1

97.Q1-11.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	-0.63*	1		
Output Gap	-0.55*	0.55*	1	
Inflation	-0.32*	0.12	0.34*	1

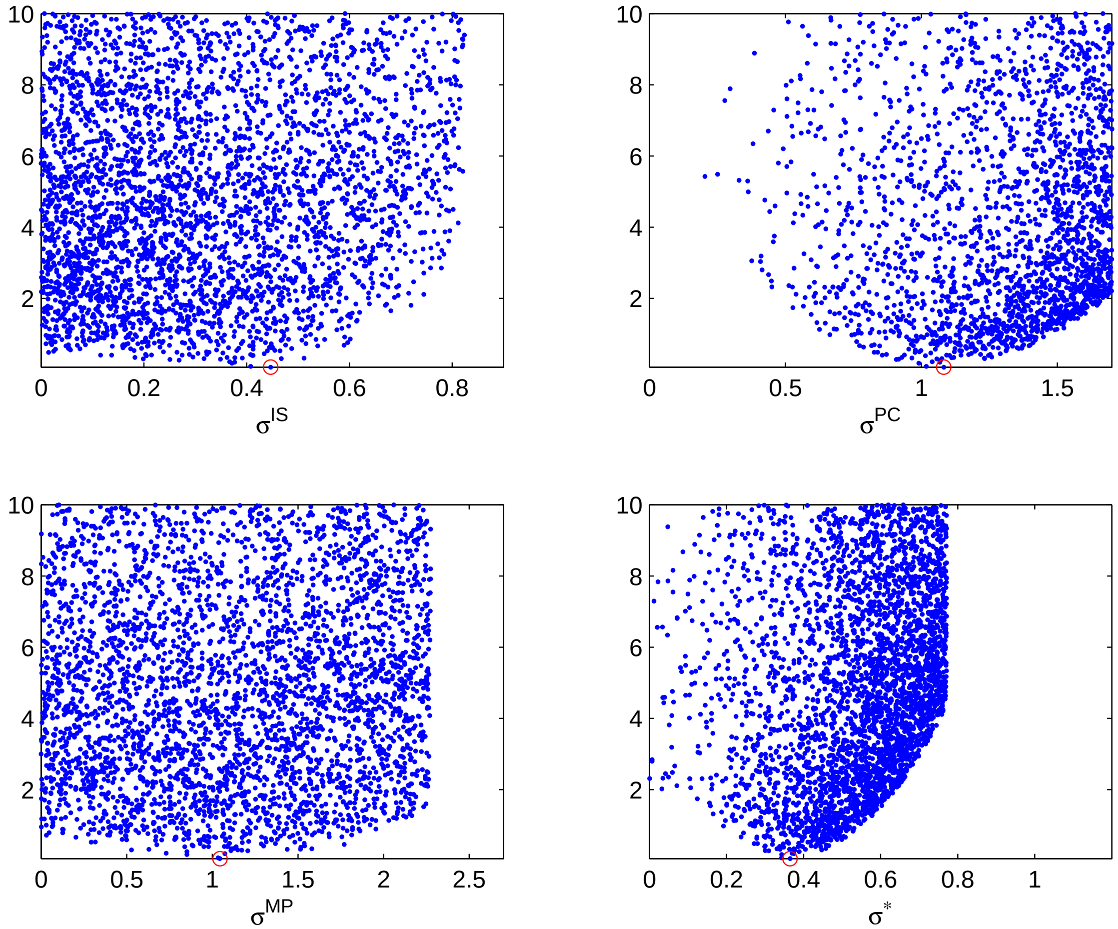
Quarterly overlapping 3 year log equity returns in excess of log three month T-bill, 3 year log excess return on 5 year nominal bond in excess of three month log T-bill. Quarterly inflation and output as in Table 1. We report correlations of log excess returns from time $t-12$ to t and macroeconomic variables as of quarter t. * and ** denote significance at the 5% and 1% level. Significance levels not adjusted for time series dependence.

Figure A.1: Minimizing with Respect to ρ^{x-} and ρ^{x+}



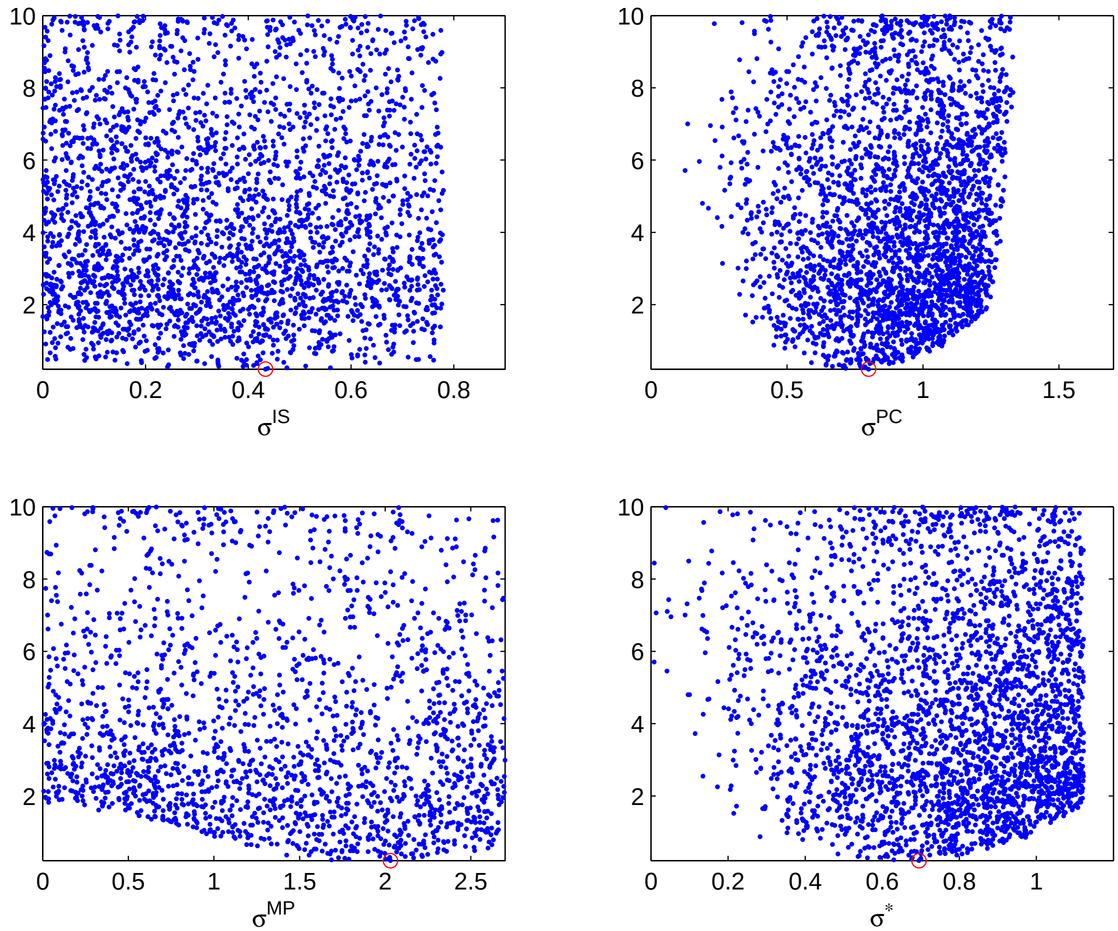
We minimize the objection function with respect to ρ^{x-} and ρ^{x+} while holding constant the volatilities of shocks at the values shown in (132). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0,1]$ and $U_2 \in [0,1]$ and set $\rho^{x-} = 0.4253 + 0.05U_1$ and $\rho^{x+} = (1 - \rho^{x-}) + 0.2 \times \rho^{x-}U_2$. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel A) Minimizing with Respect to Shock Volatilities: 1960.Q1-1979.Q2



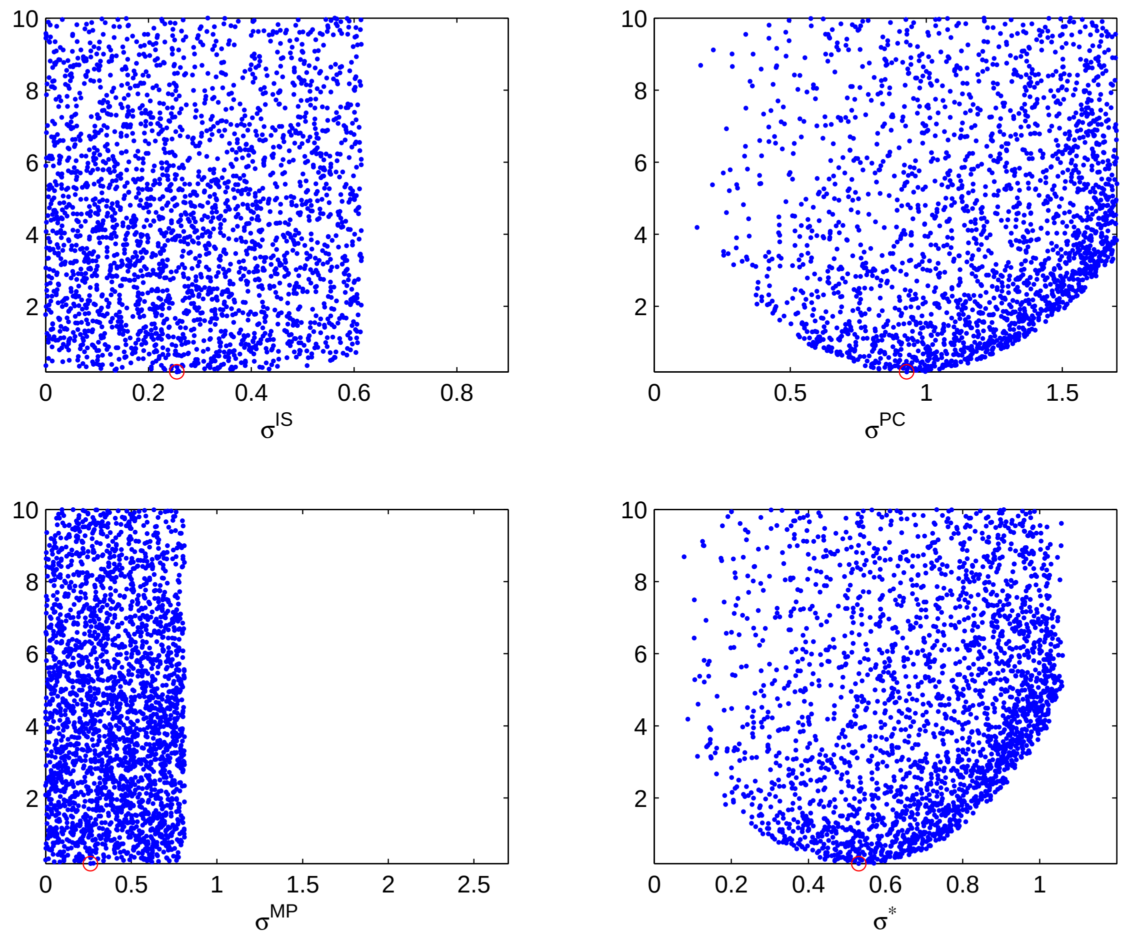
We minimize the objection function with respect to $\bar{\sigma}^{IS,1}$, $\bar{\sigma}^{PC,1}$, $\bar{\sigma}^{MP,1}$, and $\bar{\sigma}^{*,1}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel B) Minimizing with Respect to Shock Volatilities: 1979.Q3-1996.Q4



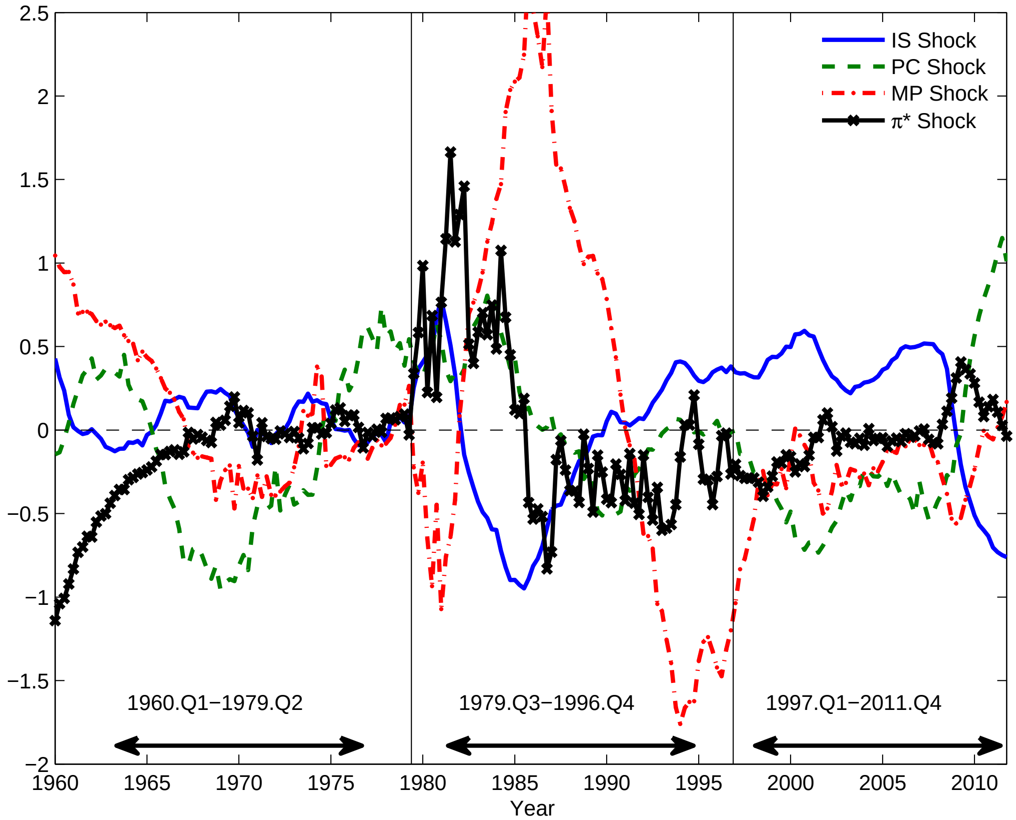
We minimize the objection function with respect to $\bar{\sigma}^{IS,2}$, $\bar{\sigma}^{PC,2}$, $\bar{\sigma}^{MP,2}$, and $\bar{\sigma}^{*,2}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel C) Minimizing with Respect to Shock Volatilities: 1997.Q1-2011.Q4



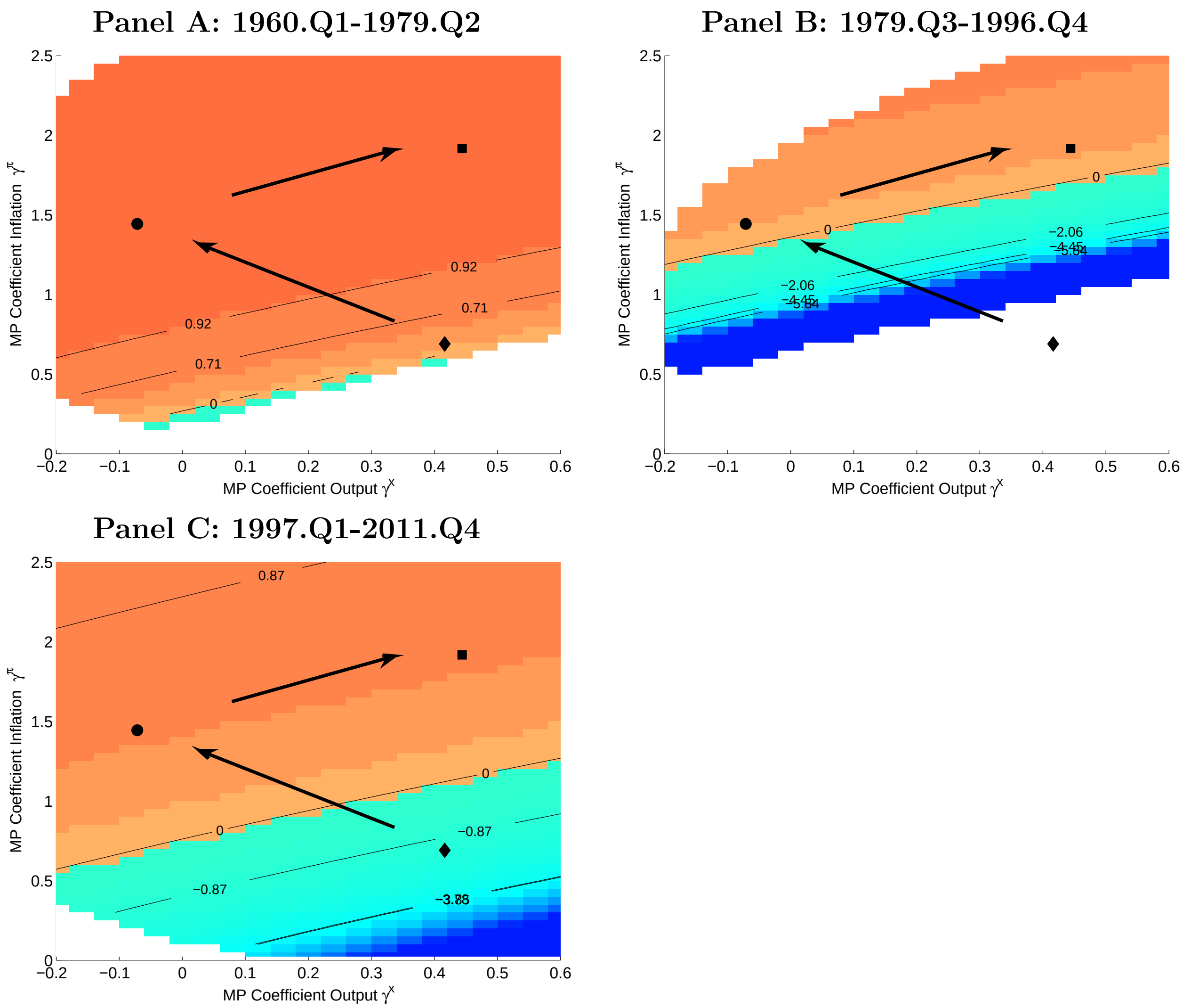
We minimize the objection function with respect to $\bar{\sigma}^{IS,3}$, $\bar{\sigma}^{PC,3}$, $\bar{\sigma}^{MP,3}$, and $\bar{\sigma}^{*,3}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper.The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.3: Time Series of Model Shocks



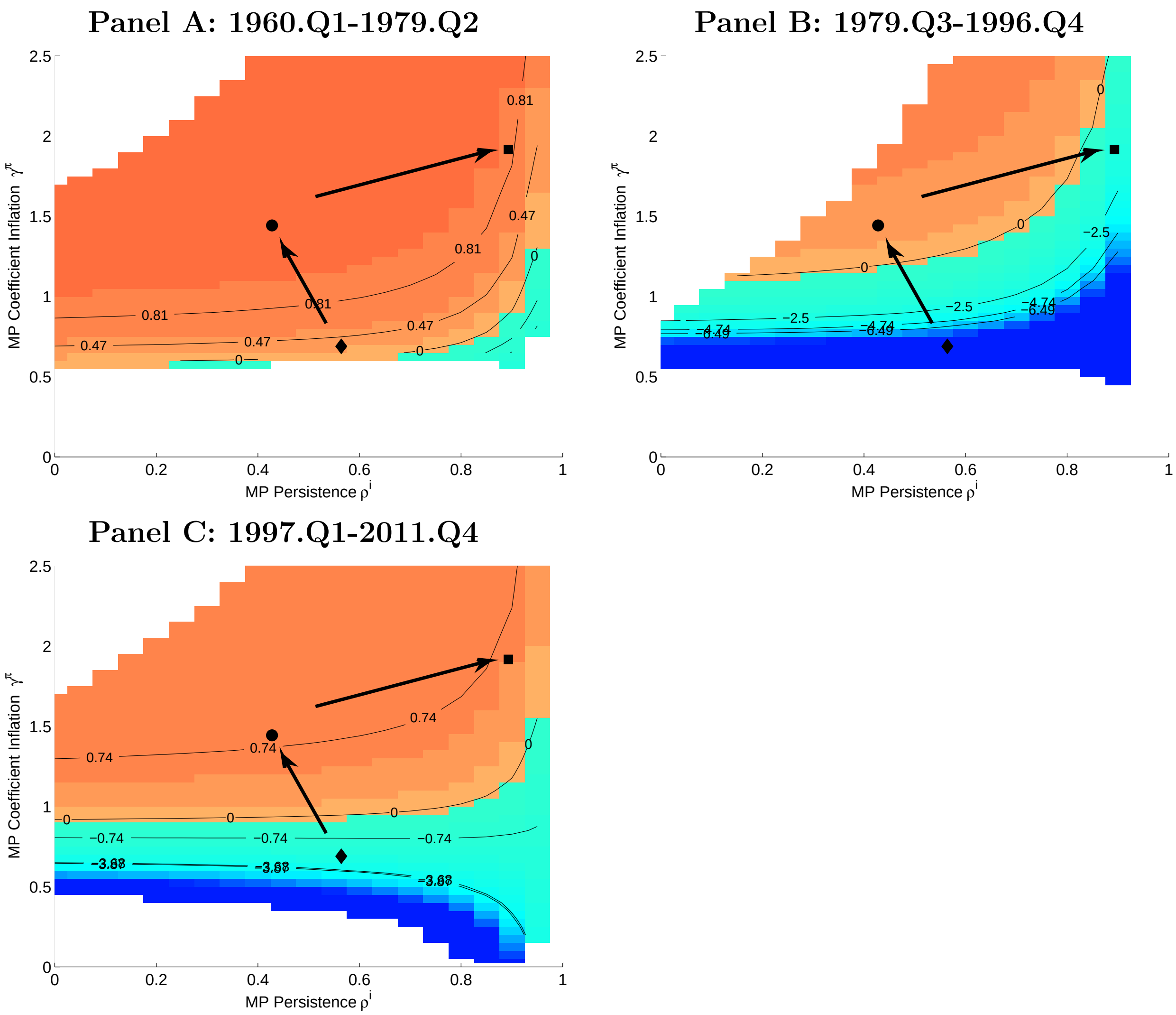
This figure plots the time series of smoothed IS, PC, MP and inflation target (π^*) shocks. IS shocks are in natural percent units, while PC, MP and inflation target shocks are in annualized percent units. The shocks are smoothed with a trailing exponentially-weighted moving average. The decay parameter equals 0.08 per quarter corresponding to a half life of 24 quarters.

Figure A.4: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and γ^{x} - Alternative Monetary Policy Rule



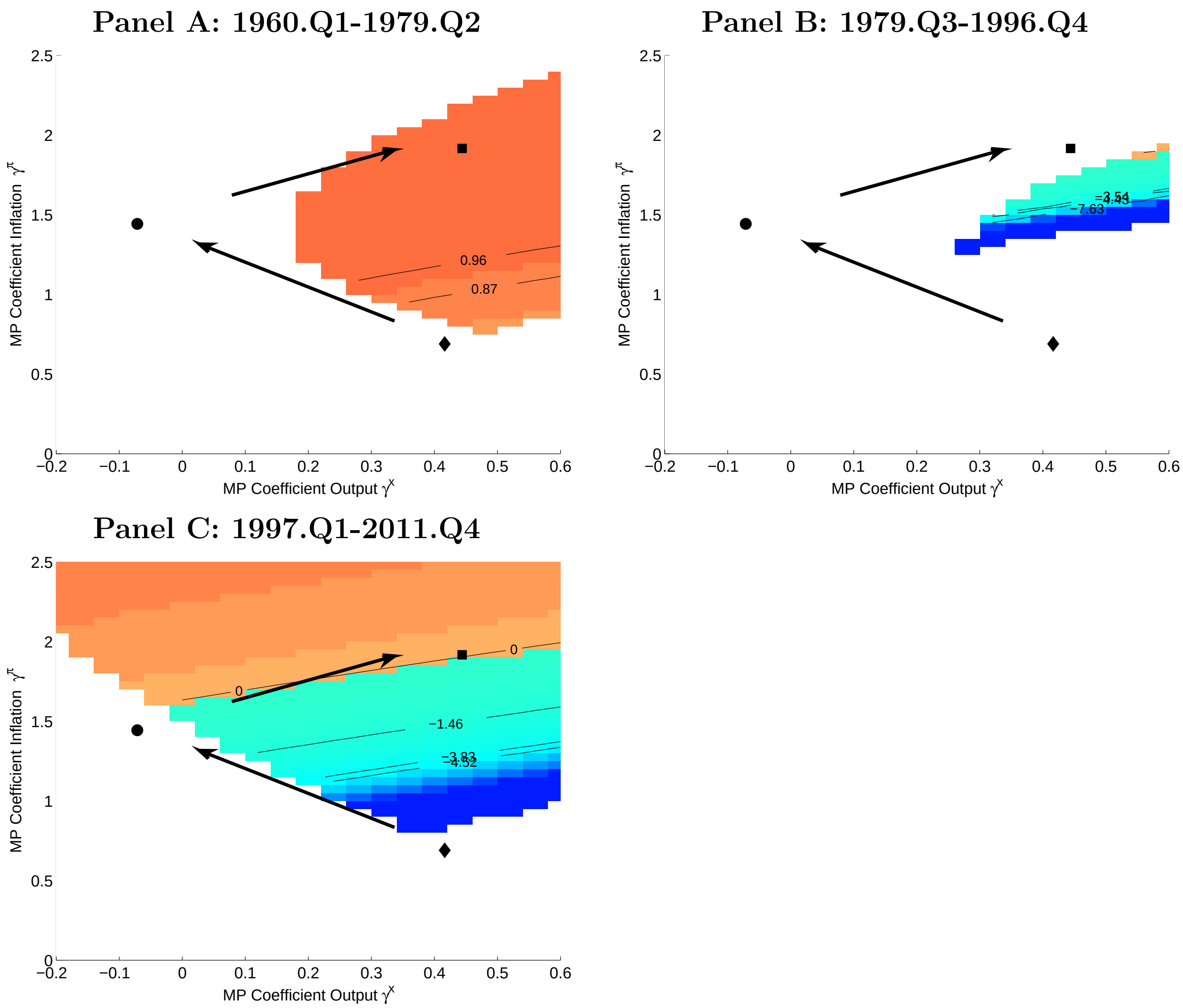
This figure re-creates Figure 3 in the main text for a model with the alternative monetary policy rule (135). All parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 3 in the main paper, because this figure was constructed with fewer pixels.

Figure A.5: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Monetary Policy Rule



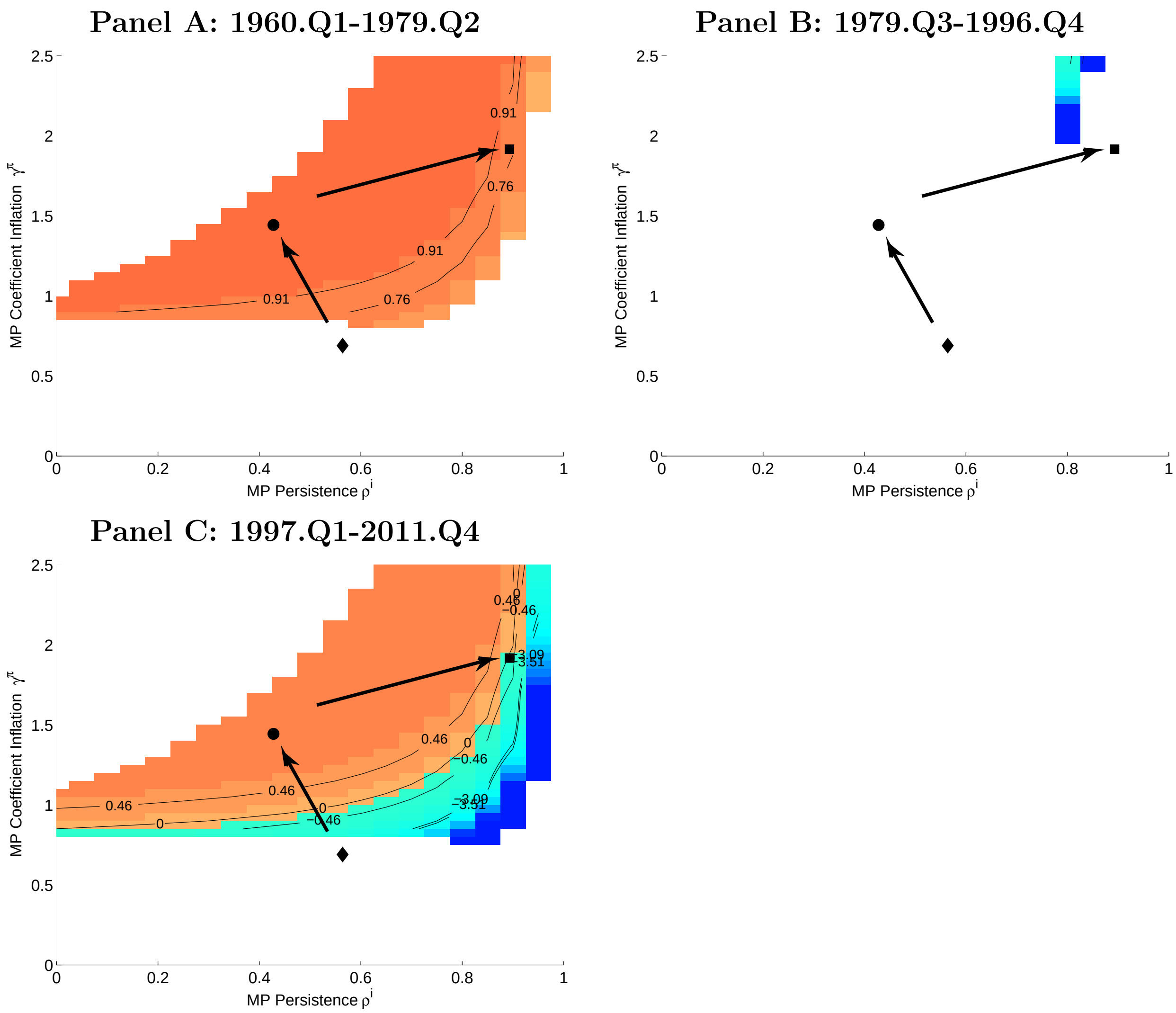
This figure re-creates Figure 4 in the main text for a model with the alternative monetary policy rule (135). All parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.6: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and γ^{x} - Alternative Phillips Curve Parameterization



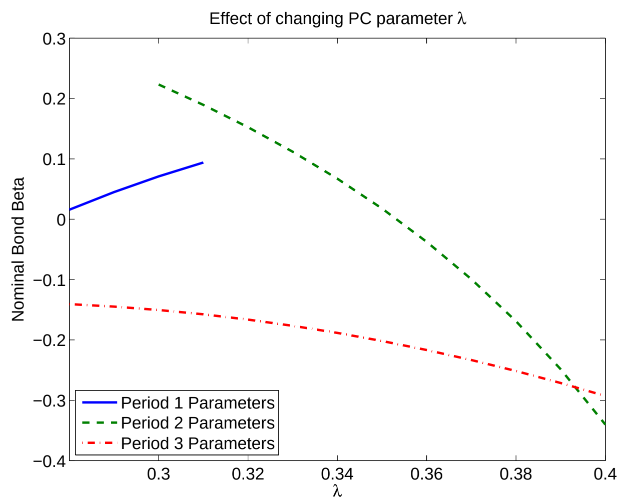
This figure re-creates Figure 3 in the main text for a model with the alternative Phillips curve parameter $\rho^{\pi} = 0.7$, which is smaller than the backward looking component in our main calibration. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 3 in the main paper, because this figure was constructed with fewer pixels.

Figure A.7: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Phillips Curve Parameterization



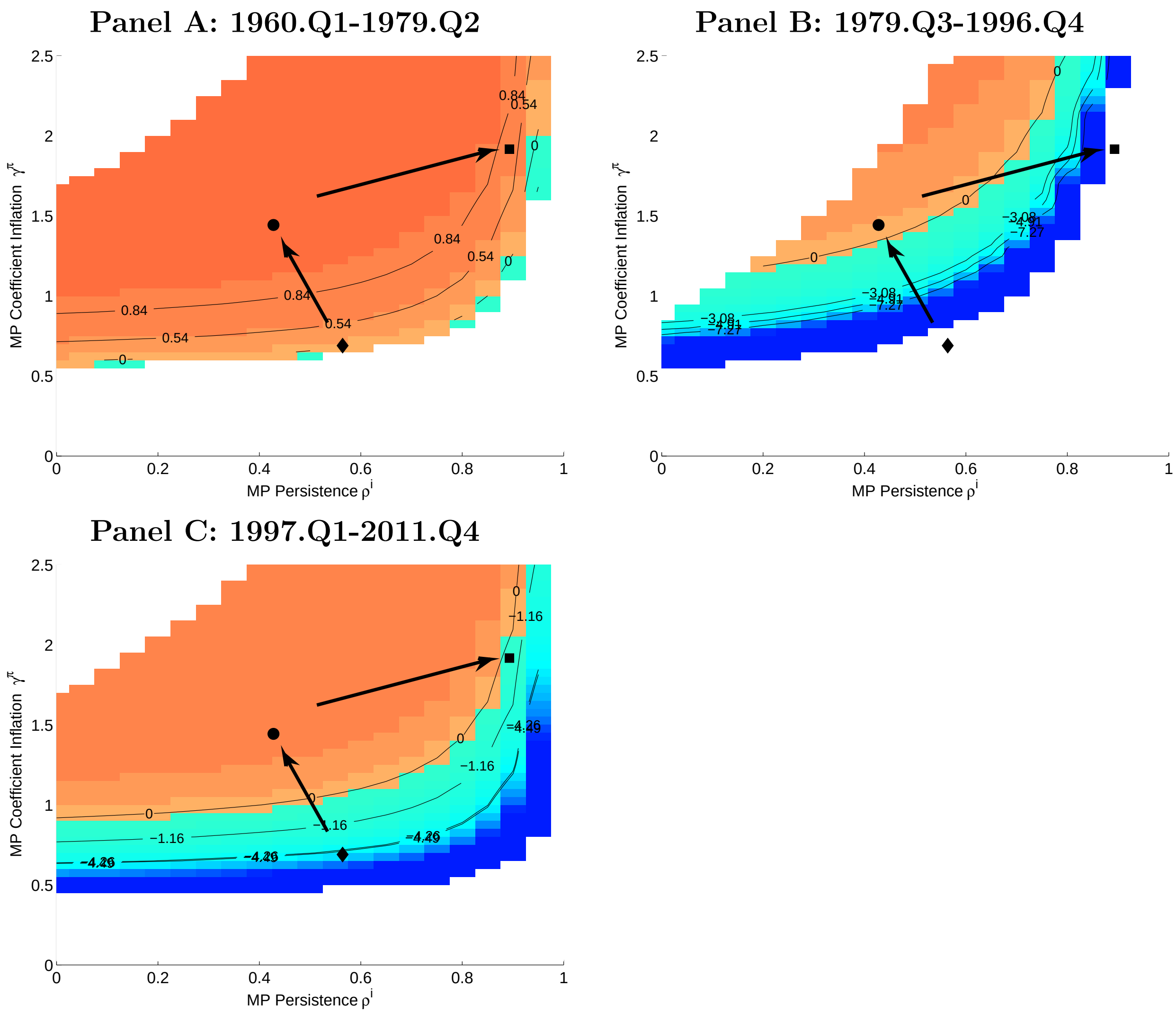
This figure re-creates Figure 4 in the main text for a model with the alternative Phillips curve parameter $\rho^{\pi} = 0.7$, which is smaller than the backward looking component in our main calibration. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.8: Nominal Bond Betas and the Phillips Curve Slope λ



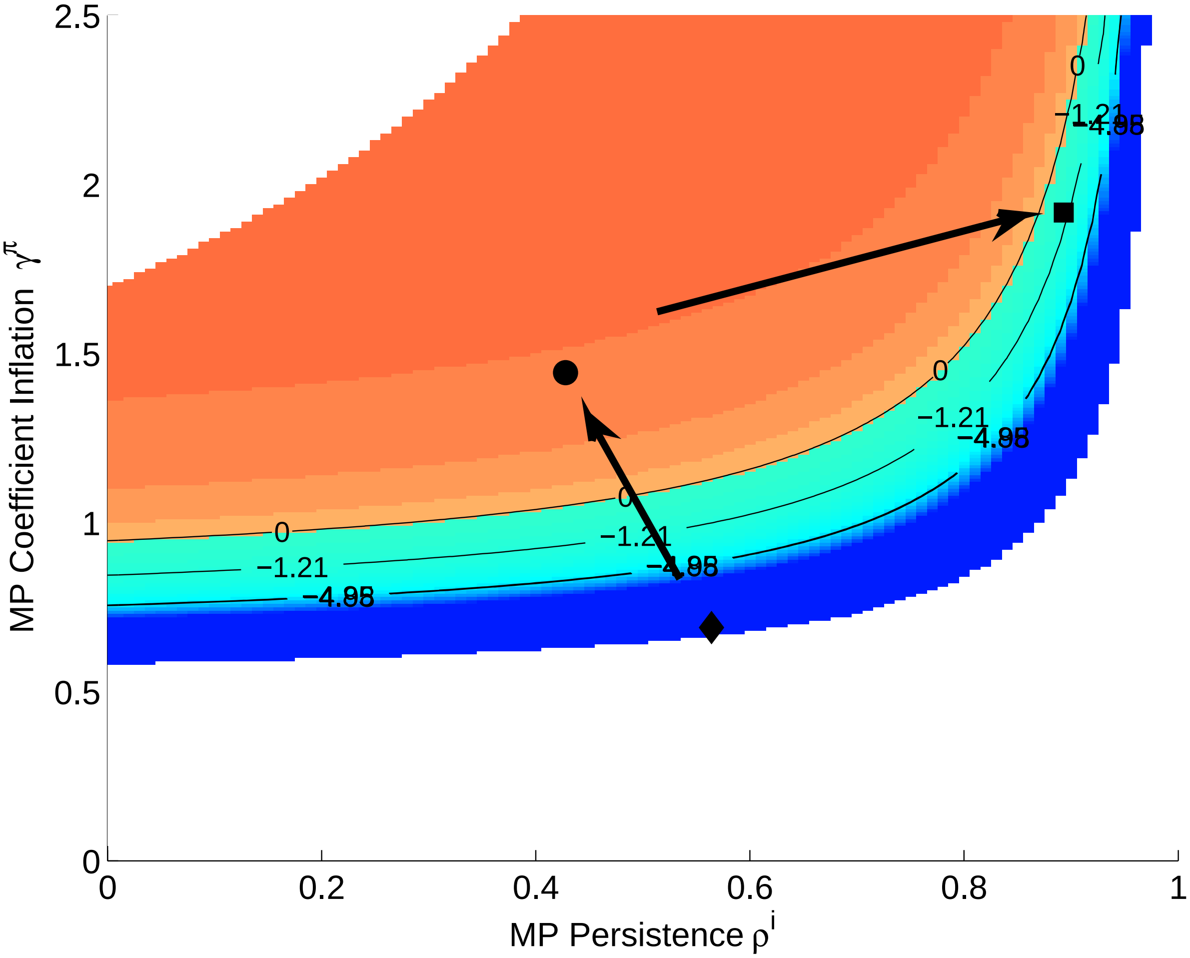
This figure shows how the nominal bond beta varies with the Phillips curve slope parameter λ while all other parameters are held constant at their period 1 values (blue solid), period 2 values (green dash), or period 3 values (red dash-dot).

Figure A.9: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Leverage Parameter



This figure re-creates Figure 4 in the main text for a model with the leverage parameter $\delta = 1$, corresponding to a firm leverage ratio of 0%. The leverage parameter in our main calibration is $\delta = 2.43$ corresponding to a firm leverage ratio of 59%. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.10: Real Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - 1960.Q1-1979.Q2 Calibration



This figure is analogous to Figure 4, Panel A in the main paper, except that this figure plots the beta of real bonds instead of nominal bonds.