Appendix for The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment

Jason Beeler and John Y. Campbell[1]

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[1]Beeler: Department of Economics, Littauer Center, Harvard University, Cambridge MA 02138, USA. Email jbeeler@fas.harvard.edu. Campbell: Department of Economics, Littauer Center, Harvard University, Cambridge MA 02138, USA, and NBER. Email john_campbell@harvard.edu.

Abstract

Section 1 of the Appendix provides solutions for the BY and BKY calibrations of the long-run risks model for the consumption claim, the dividend claim and real term structure. Section 2 describes the data used in the paper and the predictive regression used to measure the ex ante risk free rate.

1 Solving the Long-Run Risks Model

This section of the appendix provides solutions for the consumption and dividend claims for the generalized BKY endowment process

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma_{t+1}^2 &= \overline{\sigma}^2 + \nu(\sigma_t^2 - \overline{\sigma}^2) + \sigma_w w_{t+1} \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \varphi \sigma_t u_{t+1} + \pi \sigma_t \eta_{t+1} \\
& \quad w_{t+1}, e_{t+1}, u_{t+1}, \eta_{t+1} \sim i.i.d.\, \mathcal{N}(0,1).
\end{aligned}
\tag{1}
$$

In the BY model $\pi = 0$ so BY solutions are a special case. The Euler equation for the economy is

$$
\mathrm{E}_t \left[\exp \left(\theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{a,t+1} + r_{i,t+1} \right) \right] = 1, \tag{2}
$$

where $r_{a,t+1}$ is the log return on the consumption claim and $r_{i,t+1}$ is the log return on any asset. All returns are given by the approximation of Campbell and Shiller (1988) $r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{t+1,i} - z_{t,i} + \Delta d_{i,t+1}$.

Define a vector of state variables $Y_t' = [1 \;\; x_t \;\; \sigma_t^2]$ and the coefficients on the log price consumption ratio $z_t = A' Y_t$ where $A' = [A_o \;\; A_1 \;\; A_2]$. For any other asset i define coefficients in the same manner $A_i' = [A_{o,i} \;\; A_{1,i} \;\; A_{2,i}]$. This appendix prices the consumption claim and the dividend claim $z_{t,m} = A_m' Y_t$. We find z_t and $z_{t,m}$ by the method of undetermined coefficients, using the fact that the Euler equation must hold for all values of Y_t'.

The risk premium on any asset is

$$
\begin{aligned}
\mathrm{E}_t \left(r_{i,t+1} - r_{f,t} \right) + \frac{1}{2} \mathrm{Var}_t (r_{i,t+1}) &= -\mathrm{Cov}_t \left(m_{t+1}, r_{i,t+1} \right) \\
&= \sum_{j=n,e,w} \beta_{i,j} \lambda_j \sigma_{j,t}^2,
\end{aligned}
\tag{3}
$$

where $\beta_{i,j}$ is the beta and $\sigma_{j,t}^2$ the volatility of the j^{th} risk source, and the λ_j represent the price of each risk source as defined in the text.

1.1 Consumption Claim

The risk premium for the consumption claim is

$$\mathrm{E}_t\left[r_{a,t+1}-r_{f,t}\right]+\frac{1}{2}\mathrm{Var}_t\left(r_{a,t+1}\right)=\lambda_n\beta_{a,n}\sigma_t^2+\lambda_e\beta_{a,e}\sigma_t^2+\lambda_w\beta_{a,w}\sigma_w^2, \tag{4}$$

where $\beta_{a,n}=1$, $\beta_{a,e}=\kappa_1 A_1\varphi_e$ and $\beta_{a,w}=\kappa_1 A_2$. The conditional variance of the consumption claim is equal to

$$\mathrm{Var}_t\left(r_{a,t+1}\right)=\left[\beta_{a,n}^2+\beta_{a,e}^2\right]\sigma_t^2+\beta_{a,w}^2\sigma_w^2 \tag{5}$$

The coefficients A' for the log price-consumption ratio z_t are

$$\begin{aligned} A_0 &= \frac{\ln\delta+\mu_c(1-\frac{1}{\psi})+\kappa_0+\beta_{a,w}\overline{\sigma}^2\left(1-\nu\right)+\frac{1}{2}\theta\beta_{a,w}^2\sigma_w^2}{\left(1-\kappa_1\right)} \\ A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}. \\ A_2 &= \frac{\frac{1}{2}\left[\left(\theta-\frac{\theta}{\psi}\right)^2+\left(\theta\beta_{a,e}\right)^2\right]}{\theta\left(1-\kappa_1\nu_1\right)} \end{aligned} \tag{6}$$

1.2 Dividend Claim

The innovation in the market return $r_{m,t+1}-E_t(r_{m,t+1})$ is

$$r_{m,t+1}-\mathrm{E}_t(r_{m,t+1})=\varphi\sigma_t u_{t+1}+\beta_{m,\eta}\sigma_t\eta_{t+1}+\beta_{m,e}\sigma_t e_{t+1}+\beta_{m,w}\sigma_w w_{t+1}, \tag{7}$$

where $\beta_{m,\eta}=\pi$, $\beta_{m,e}=\kappa_{1,m}A_{1,m}\varphi_e$ and $\beta_{m,w}=\kappa_{1,m}A_{2,m}$, which implies that the risk premium on the dividend claim is

$$\mathrm{E}_t\left[r_{m,t+1}-r_{f,t}\right]+\frac{1}{2}\mathrm{Var}_t\left(r_{m,t+1}\right)=\lambda_\eta\beta_{m,\eta}\sigma_t^2+\lambda_e\beta_{m,e}\sigma_t^2+\lambda_w\beta_{m,w}\sigma_w^2. \tag{8}$$

In the BY calibration $\pi = 0$ so the premium from consumption shocks is zero. The coefficients A_m' for the log price-dividend ratio are as follows

$$
\begin{aligned}
A_{0,m} &= \frac{\left[\begin{array}{c} \theta\log(\delta) + \mu_c(\theta - \frac{\theta}{\psi} - 1) - \lambda_w \overline{\sigma}^2(1-\nu) + (\theta - 1)\left[\kappa_0 + A_0\left(\kappa_1 - 1\right)\right] \\ +\kappa_{0,m} + \beta_{m,w}\overline{\sigma}^2(1-\nu) + \mu_d + \frac{1}{2}\left[\beta_{m,w} - \lambda_w\right]^2 \sigma_w^2 \end{array}\right]}{(1-\kappa_{1,m})} \\
A_{1,m} &= \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m}\rho}. \\
A_{2,m} &= \frac{(1-\theta)\,A_2(1-\kappa_1 \upsilon_1) + \frac{1}{2}\left[(\pi - \lambda_n)^2 + \left(\beta_{m,e} - \lambda_e\right)^2 + \varphi^2\right]}{(1 - \kappa_{1,m}\nu)}
\end{aligned} \tag{9}
$$

1.3 Risk Free Interest Rate

To derive the risk free rate, we use the Euler equation for a riskless asset:

$$
\begin{aligned}
r_{f,t+1} &= -\theta\log(\delta) + \frac{\theta}{\psi}\mathrm{E}_t\left[\Delta c_{t+1}\right] + (1-\theta)\,\mathrm{E}_t r_{a,t+1} \\
&\quad -\frac{1}{2}\mathrm{Var}_t\left[\frac{\theta}{\psi}\Delta c_{t+1} + (1-\theta)\,r_{a,t+1}\right].
\end{aligned} \tag{10}
$$

We subtract $(1-\theta)\,r_{f,t+1}$ from both sides and divide by θ, assuming $\theta \neq 0$. It follows that:

$$
\begin{aligned}
r_{f,t+1} &= -\log(\delta) + \frac{1}{\psi}\mathrm{E}_t\left[\Delta c_{t+1}\right] + \frac{(1-\theta)}{\theta}\mathrm{E}_t\left[r_{a,t+1} - r_{f,t}\right] \\
&\quad -\frac{1}{2\theta}\mathrm{Var}_t\left(m_{t+1}\right),
\end{aligned} \tag{11}
$$

$\mathrm{Var}_t\left(m_{t+1}\right) = \left(\lambda_n^2 + \lambda_e^2\right)\sigma_t^2 + \lambda_w^2\sigma_w^2$ and $\mathrm{E}_t\left[r_{a,t+1} - r_{f,t}\right]$ is given above.

1.4 Linearization Parameters

For any asset, the linearization parameters are determined endogenously by the following system of equations as discussed in Campbell and Koo (1997) and Bansal, Kiku and Yaron (2007):

$$\begin{aligned}
\overline{z_i} &= A_{0,i}(\overline{z_i}) + A_{2,i}(\overline{z_i})\sigma^2 \\
\kappa_{1,i} &= \frac{\exp(\overline{z_i})}{1+\exp(\overline{z_i})} \\
\kappa_{0,i} &= \ln(1+\exp(\overline{z_i})) - \kappa_{1,i}\overline{z_i}
\end{aligned} \tag{12}$$

The solution is determined numerically by iteration until reaching a fixed point of $\overline{z_i}$. The dependence of $A_{0,i}$ and $A_{2,i}$ on the linearization parameters is discussed in the previous sections.

1.5 Zero Coupon Term Structure

We conjecture that the log price of a zero coupon bond of maturity n at time t is equal to

$$p_{n,t} = -B_{0,n} - B_{x,n}x_t - B_{\sigma,n}\sigma_t^2$$

and solve for the coefficients B_0, B_x and B_σ by recursion. The log bond price for a one period risk free bond is just the opposite of the risk free rate

$$p_{1,t} = -y_{1,t} = -r_{f,t+1}$$

where $y_{n,t}$ is the log yield of maturity n at time t. This provides the solutions for $B_{0,1}$, $B_{x,1}$ and $B_{\sigma,1}$ implicitly. In addition, the consumer's Euler equation links the bond price in periods t and $t+1$

$$P_{n,t} = E_t\left[M_{t+1}P_{n-1,t+1}\right]$$

Because the bond price and stochastic discount factor are jointly loglinear, the log price $p_{n,t}$ is

$$p_{n,t} = E_t\left(m_{t+1} + p_{n-1,t+1}\right) + \frac{1}{2}Var_t\left(m_{t+1} + p_{n-1,t+1}\right)$$

which allows one to solve the recursion for the coefficients B sequentially for all values of n. To calculate the return of buying a zero coupon bond in one month and selling it the next, we use

$$R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$$

2 Data

2.1 Consumption

Consumption data is from the Bureau of Economic Analysis. We use series 7.1: real nondurable consumption per capita and real services consumption per capita. These series are available at both annual and quarterly frequency. Define $C_{nd,\,t}$ as real nondurables and $C_{s\,,t}$ as real services consumption per capita. Then the growth rate of consumption in year or quarter $t+1$ is

$$\Delta c_{t+1} = \ln\left[\frac{C_{nd,t+1} + C_{s,t+1}}{C_{nd,\,t} + C_{s\,,t}}\right]$$

Because real consumption data is chain weighted, this is not an exact formula for the true growth rate of nondurables and services consumption. We also calculated consumption growth using the Tornqvist index method, which gives a better approximation to the true growth rate of consumption. This method treats the growth in real consumption as a weighted average of the growth rates of real nondurable and services consumption, with the weights determined each period based on nominal consumption. The results reported in the paper were similar to those with the Tornqvist index calculation.

2.2 Stock Market Data

Two series from the CRSP are used, the value-weighted index including distributions (VWRETD) and the value-weighted index excluding distributions (VWRETX). A monthly price index for the market is constructed as

$$P_{s+1} = P_s\left(1 + VWRETX_{s+1}\right)$$

And the monthly dividend is given by

$$D_{s+1} = P_{s+1}\left[\frac{(1 + VWRETD_{s+1})}{(1 + VWRETX_{s+1})} - 1\right]$$

In order to calculate returns, we first calculate simple monthly returns, then take logs and add monthly returns within a quarter or year. The yearly or quarterly dividend is the sum of dividends within a year or quarter. We then calculate the log year over year or quarter over quarter growth rate in dividends. Quarterly dividends are not seasonally adjusted. The price-dividend ratio is the price in the last month of the year or quarter divided by the sum of dividends paid in the last twelve months.

2.3 Inflation

Both stock returns and dividend growth are converted from nominal to real terms using the CPI from the Bureau of Labor Statistics. For yearly inflation, we use the seasonally unadjusted CPI from the BLS. Yearly inflation is the log December over December growth rate in the CPI. For quarterly inflation, we use seasonally adjusted CPI. Similarly, quarterly inflation is the log growth rate of the CPI in the final month of the current quarter over the final month in the previous quarter. For both dividends and stock returns, we subtract log inflation to form real growth rates or returns.

2.4 Ex Ante Risk Free Rate

Nominal yields to calculate risk free rates are the CRSP Fama Risk Free Rates. We use the three month yield even though agents in the model have a monthly time interval because of the larger volume and higher reliability of three month Treasury bills.

We take the nominal log yield on a three month Treasury bill $y_{3,t}$ in month t and subtract three month log inflation $\pi_{t,t+3}$ from period t to $t+3$ to form a measure of the ex post real three month interest rate. This is the dependent variable in the predictive regression below. The independent variables are average quarterly log inflation over the previous year $\pi_{t-12,t}$ (annual log inflation divided by four) and the three month nominal yield $y_{3,t}$

$$y_{3,t} - \pi_{t,t+3} = \beta_0 + \beta_1 y_{3,t} + \beta_2 \pi_{t-12,t} + \varepsilon_{t+3}$$

Because both inflation and bond yields are observed monthly, we use a monthly time interval for the regression. The monthly sample for the yearly data is 1929.12-2008.12 and for quarterly data is 1947.03-2008.12. The predicted value for the regression is the ex ante risk free rate

$$\widehat{rf}_{t+1} = \widehat{\beta}_0 + \widehat{\beta}_1 y_{3,t} + \widehat{\beta}_2 \pi_{t-12,t}$$

For the yearly data, we use the annualized quarterly value of rf_{t+1} at the beginning of the year as the yearly risk free rate. This has the advantage that all of the variables used to predict the risk free rate in the regression are known at the beginning of the year. An alternative method would be to add quarterly log ex ante risk free rates over the year to form an annual risk free rate. This has the advantage of being closer to an implementable investment strategy of rolled over treasury bills. For the quarterly data, the ex ante risk free rate is just the predicted value at the beginning of the quarter.

This regression is equivalent to a model forecasting three month ahead inflation

$$\pi_{t,t+3} = \alpha_0 + \alpha_1 y_{3,t} + \alpha_2 \pi_{t-12,t} + \zeta_{t+3}$$

where the following restrictions are imposed

$$\begin{aligned} \beta_0 &= -\alpha_0 \\ \beta_1 &= 1 - \alpha_1 \\ \beta_2 &= -\alpha_2 \end{aligned}$$

For the quarterly sample, seasonally adjusted CPI data is available to calculate the quarterly inflation used for the dependent variable. Since the seasonally adjusted CPI data begins in 1947, the observations for $\pi_{t-12,t}$ are calculated using seasonally unadjusted CPI, but this doesn't matter because it is year over year inflation.

Since there is no seasonally adjusted data before 1947, calculating $\pi_{t,t+3}$ is more difficult in the annual data. However, while $\pi_{t,t+3}$ displays seasonality, there are no seasonal patterns in the independent variables to fit the seasonal pattern in three month inflation. Since the measurement error that arises from seasonality is in the dependent variable and none of the independent variables are correlated with the seasonal pattern, the point estimates in the inflation forecasting regression should be an unbiased forecast of seasonally adjusted inflation. To test this, we estimate the model with seasonally unadjusted CPI and seasonally adjusted CPI, with the adjustment done manually by X-12 ARIMA. The coefficients are extremely close to one another regardless of whether the seasonal adjustment is used.

References

Bansal, Ravi, Dana Kiku and Amir Yaron, 2007, "Risks for the Long Run: Estimation and Inference", unpublished paper, Duke University and University of Pennsylvania.

Campbell, John Y. and Hyeng Keun Koo, 1997, "A Comparison of Numerical and Analytical Approximate Solutions to an Intertemporal Consumption Choice Problem", *Journal of Economic Dynamics and Control* 21, 273–295.

Appendix Table I

Autocorrelations of Consumption and Dividends

Consumption Autocorrelations

Moment	$\widehat{\boldsymbol{\rho}}$ **data**	$\boldsymbol{\rho}(50\%)$ **BY**	$\boldsymbol{\rho}(50\%)$ **BKY**	$\%\,(\widehat{\boldsymbol{\rho}})$ **BY**	$\%\,(\widehat{\boldsymbol{\rho}})$ **BKY**
$AC1$ 1930-2008	0.451	0.469	0.398	0.436	0.679
$AC2$ 1930-2008	0.156	0.226	0.148	0.311	0.523
$AC3$ 1930-2008	-0.097	0.157	0.097	**0.037**	0.084
$AC4$ 1930-2008	-0.240	0.105	0.058	**0.006**	**0.013**
$AC5$ 1930-2008	-0.020	0.066	0.031	0.267	0.351
$AC1$ 1948-2008	0.321	0.457	0.387	0.154	0.312
$AC2$ 1948-2008	-0.017	0.209	0.134	0.084	0.174
$AC3$ 1948-2008	-0.071	0.139	0.081	0.093	0.161
$AC4$ 1948-2008	-0.010	0.087	0.044	0.264	0.359
$AC5$ 1948-2008	-0.007	0.049	0.018	0.354	0.433

Dividend Autocorrelations

Moment	$\widehat{\boldsymbol{\rho}}$ **data**	$\boldsymbol{\rho}(50\%)$ **BY**	$\boldsymbol{\rho}(50\%)$ **BKY**	$\%\,(\widehat{\boldsymbol{\rho}})$ **BY**	$\%\,(\widehat{\boldsymbol{\rho}})$ **BKY**
$AC1$ 1930-2008	0.210	0.366	0.260	0.088	0.333
$AC2$ 1930-2008	-0.198	0.122	0.013	**0.007**	**0.050**
$AC3$ 1930-2008	-0.139	0.083	0.004	**0.043**	0.126
$AC4$ 1930-2008	-0.145	0.055	-0.001	0.057	0.125
$AC5$ 1930-2008	0.020	0.032	-0.008	0.465	0.592
$AC1$ 1948-2008	0.315	0.354	0.252	0.380	0.692
$AC2$ 1948-2008	0.112	0.106	0.005	0.515	0.768
$AC3$ 1948-2008	-0.087	0.069	-0.001	0.143	0.270
$AC4$ 1948-2008	-0.174	0.040	-0.008	0.063	0.113
$AC5$ 1948-2008	-0.015	0.019	-0.013	0.401	0.496

Table I displays consumption and dividend autocorrelations in yearly data and for the BY and BKY calibrations. The consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to yearly levels, then computing the growth rate, then taking logs. The second column displays the moment in the data, the next two display the medians for the two calibrations, followed by the percentile of the data moment in both calibrations. The results are displayed both for the full 1930-2008 sample and a postwar sample of 1948-2008. The medians are from 100,000 samples of equivalent length to the data (948 or 732 months) and the percentile is the proportion of those samples with an estimate at or below that of the data. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Appendix Table II
Regression Coefficients for Predictability by the Price-Dividend Ratio

$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	-0.093	-0.040	-0.090	0.310	0.490
3 Y	-0.264	-0.117	-0.258	0.293	0.491
5 Y	-0.413	-0.190	-0.406	0.287	0.494
4 Q	-0.119	-0.051	-0.101	0.284	0.446
12 Q	-0.274	-0.150	-0.285	0.340	0.512
20 Q	-0.424	-0.241	-0.442	0.341	0.514

$\sum_{j=1}^{J}(\Delta c_{t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	0.011	0.086	0.051	**0.000**	**0.045**
3 Y	0.010	0.206	0.097	**0.000**	0.090
5 Y	-0.001	0.265	0.116	**0.002**	0.125
4 Q	0.000	0.083	0.040	**0.000**	0.062
12 Q	-0.002	0.185	0.079	**0.003**	0.125
20 Q	-0.003	0.230	0.092	**0.014**	0.195

$\sum_{j=1}^{J}(\Delta d_{t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	0.074	0.381	0.332	**0.000**	**0.017**
3 Y	0.107	0.735	0.435	**0.001**	0.132
5 Y	0.089	0.907	0.472	**0.006**	0.199
4 Q	0.003	0.276	0.146	**0.001**	0.119
12 Q	0.012	0.570	0.222	**0.013**	0.262
20 Q	0.044	0.695	0.235	**0.048**	0.352

Column 2 of Table II displays coefficients from predictive regressions of excess returns, consumption growth and dividend growth on log price-dividend ratios in the 1930-2008 annual and 1947.2-2008.4 quarterly datasets. Throughout the table, the first part of each panel dsiplays annual results and the second quarterly. The next two columns following the data moments display the median coefficient from finite sample simulations of the two calibrations. The last two columns report the percentile of the data moment for the model in both calibrations. Standard errors are Newey-West with 2*(horizon-1) lags. The medians from 100,000 samples of equivalent length to the data (948 or 741 months) and the percentile is the proportion of those samples with an estimate at or below that of the data. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Appendix Table III
Coefficients for Predictability of Volatility

Excess Return Volatility

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	-0.084	-0.092	-0.943	0.505	0.844
3 Y	-0.027	-0.073	-0.833	0.539	0.901
5 Y	0.017	-0.060	-0.733	0.575	0.912
4 Q	0.056	-0.087	-0.899	0.680	**0.969**
12 Q	0.149	-0.071	-0.770	0.790	**0.985**
20 Q	0.161	-0.057	-0.656	0.807	**0.984**

Consumption Volatility

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	-0.697	-0.092	-1.039	0.176	0.643
3 Y	-0.583	-0.082	-0.957	0.127	0.683
5 Y	-0.560	-0.068	-0.853	0.106	0.657
4 Q	-0.619	-0.091	-1.014	**0.045**	0.671
12 Q	-0.641	-0.074	-0.872	**0.022**	0.610
20 Q	-0.552	-0.060	-0.744	**0.029**	0.606

Dividend Volatility

	$\widehat{\beta}$ *data*	$\beta\,(50\%)$ *BY*	$\beta\,(50\%)$ *BKY*	$\%\left(\widehat{\beta}\right)$ *BY*	$\%\left(\widehat{\beta}\right)$ *BKY*
1 Y	-0.501	-0.119	-1.111	0.270	0.749
3 Y	-0.248	-0.094	-0.982	0.361	0.851
5 Y	-0.148	-0.077	-0.869	0.428	0.876
4 Q	-0.370	-0.091	-1.035	0.182	0.812
12 Q	-0.247	-0.074	-0.880	0.260	0.852
20 Q	-0.189	-0.059	-0.746	0.302	0.855

Columns 2 of Table III displays coefficients from predictive regressions of excess return, consumption or dividend volatility on the log price-dividend ratio for the 1930-2008 annual and 1947.2-2008.4 quarterly datasets. Volatility is measured as the sum of absolute residulas from an AR(1) model of consumption growth, dividend growth or excess returns. Throughout the table, the first part of each panel dsiplays annual results and the second quarterly. The next two columns following the data moments display the median regression coefficients from finite sample simulations of the two calibrations. The last two columns report the percentile of the data moment for the model in both calibrations. Standard errors are Newey-West with 2*(horizon-1) lags. The medians from 100,000 samples of equivalent length to the data (948 or 741 months) and the percentile is the proportion of those samples with an estimate at or below that of the data. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Appendix Table IVA
Multivariate Predictability: Coefficients BY Model

$\sum_{j=1}^{J}\left(r_{m,t+j}-r_{f,t+j}\right)=\alpha+\beta_1\left(r_{m,t}-r_{f,t}\right)+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.066	-0.096	-0.328
1 Y	$\beta\,(50\%)$	0.006	-0.040	-0.239
	$\%\left(\widehat{\beta}\right)$	0.600	0.444	0.493
	$\widehat{\beta}$	-0.023	-0.261	-0.092
3 Y	$\beta\,(50\%)$	0.001	-0.101	-0.684
	$\%\left(\widehat{\beta}\right)$	0.476	0.411	0.527
	$\widehat{\beta}$	-0.057	-0.402	-0.333
5 Y	$\beta\,(50\%)$	-0.005	-0.148	-1.173
	$\%\left(\widehat{\beta}\right)$	0.461	0.390	0.530

$\sum_{j=1}^{J}\left(\Delta c_{t+j}\right)=\alpha+\beta_1\Delta c_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.399	0.008	-0.084
1 Y	$\beta\,(50\%)$	0.204	0.042	0.518
	$\%\left(\widehat{\beta}\right)$	**0.965**	0.112	0.096
	$\widehat{\beta}$	0.502	0.005	0.039
3 Y	$\beta\,(50\%)$	0.125	0.123	1.261
	$\%\left(\widehat{\beta}\right)$	0.884	**0.025**	0.156
	$\widehat{\beta}$	0.335	-0.008	0.206
5 Y	$\beta\,(50\%)$	0.046	0.168	1.563
	$\%\left(\widehat{\beta}\right)$	0.723	**0.023**	0.238

$\sum_{j=1}^{J}\left(\Delta d_{t+j}\right)=\alpha+\beta_1\Delta d_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.110	0.086	-1.140
1 Y	$\beta\,(50\%)$	0.104	0.560	-3.693
	$\%\left(\widehat{\beta}\right)$	0.525	**0.000**	0.947
	$\widehat{\beta}$	-0.274	0.134	-1.735
3 Y	$\beta\,(50\%)$	-0.027	0.796	-1.196
	$\%\left(\widehat{\beta}\right)$	0.159	**0.001**	0.451
	$\widehat{\beta}$	-0.389	0.113	-1.289
5 Y	$\beta\,(50\%)$	-0.127	0.919	-0.040
	$\%\left(\widehat{\beta}\right)$	0.233	**0.002**	0.427

$\sum_{j=1}^{J}\left(r_{f,t+j}\right)=\alpha+\beta_1\Delta c_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	-0.356	0.006	0.570
1 Y	$\beta\,(50\%)$	-0.011	0.038	0.333
	$\%\left(\widehat{\beta}\right)$	**0.000**	**0.000**	**0.974**
	$\widehat{\beta}$	-0.900	0.022	1.041
3 Y	$\beta\,(50\%)$	-0.032	0.088	0.702
	$\%\left(\widehat{\beta}\right)$	**0.000**	**0.005**	0.750
	$\widehat{\beta}$	-0.576	0.034	1.720
5 Y	$\beta\,(50\%)$	-0.049	0.115	0.808
	$\%\left(\widehat{\beta}\right)$	**0.023**	**0.032**	0.833

Table IVA displays coefficients for predictive regressions of excess returns, consumption growth, dividend growth or the risk free rate on predictor variables in the 1930-2008 annual dataset. For consumption growth, dividend growth and excess returns the predictor variables are the risk free rate, the log price-dividend ratio and lagged consumption growth, dividend growth or excess returns. For the risk free rate, the predictor variables are the log price-dividend ratio, consumption growth and the lagged risk free rate. The upper left panel displays the results for excess returns, the upper right for consumption growth, the bottom left for dividend growth and the bottom right for risk free rates. For each coefficients, the median from 100,000 simulations of the BY model is displayed below. The percentile of the finite sample simulations corresponding to the coefficient is under the median. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Appendix Table IVB
Multivariate Predictability: Coefficients BKY Model

$\sum_{j=1}^{J}\left(r_{m,t+j}-r_{f,t+j}\right)=\alpha+\beta_1\left(r_{m,t}-r_{f,t}\right)+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.066	-0.096	-0.328
1 Y	$\beta\,(50\%)$	0.033	-0.133	0.461
	$\%\left(\widehat{\beta}\right)$	0.584	0.569	0.412
	$\widehat{\beta}$	-0.023	-0.261	-0.092
3 Y	$\beta\,(50\%)$	0.079	-0.369	1.321
	$\%\left(\widehat{\beta}\right)$	0.376	0.578	0.438
	$\widehat{\beta}$	-0.057	-0.402	-0.333
5 Y	$\beta\,(50\%)$	0.119	-0.568	1.956
	$\%\left(\widehat{\beta}\right)$	0.355	0.576	0.430

$\sum_{j=1}^{J}\left(\Delta c_{t+j}\right)=\alpha+\beta_1\Delta c_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.399	0.008	-0.084
1 Y	$\beta\,(50\%)$	0.214	0.019	0.703
	$\%\left(\widehat{\beta}\right)$	0.940	0.326	**0.037**
	$\widehat{\beta}$	0.502	0.005	0.039
3 Y	$\beta\,(50\%)$	0.126	0.012	2.280
	$\%\left(\widehat{\beta}\right)$	0.868	0.450	**0.046**
	$\widehat{\beta}$	0.335	-0.008	0.206
5 Y	$\beta\,(50\%)$	0.042	0.011	2.934
	$\%\left(\widehat{\beta}\right)$	0.730	0.425	0.095

$\sum_{j=1}^{J}\left(\Delta d_{t+j}\right)=\alpha+\beta_1\Delta d_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	0.110	0.086	-1.140
1 Y	$\beta\,(50\%)$	0.137	0.389	-2.274
	$\%\left(\widehat{\beta}\right)$	0.415	**0.021**	0.673
	$\widehat{\beta}$	-0.274	0.134	-1.735
3 Y	$\beta\,(50\%)$	0.050	0.360	1.880
	$\%\left(\widehat{\beta}\right)$	0.105	0.255	0.301
	$\widehat{\beta}$	-0.389	0.113	-1.289
5 Y	$\beta\,(50\%)$	-0.028	0.347	3.673
	$\%\left(\widehat{\beta}\right)$	0.140	0.327	0.319

$\sum_{j=1}^{J}\left(r_{f,t+j}\right)=\alpha+\beta_1\Delta c_t+\beta_2\left(p_t-d_t\right)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

		β_1	β_2	β_3
	$\widehat{\beta}$	-0.356	0.006	0.570
1 Y	$\beta\,(50\%)$	-0.017	0.012	0.670
	$\%\left(\widehat{\beta}\right)$	**0.000**	0.136	0.185
	$\widehat{\beta}$	-0.900	0.022	1.041
3 Y	$\beta\,(50\%)$	-0.051	0.035	1.370
	$\%\left(\widehat{\beta}\right)$	**0.000**	0.274	0.246
	$\widehat{\beta}$	-0.576	0.034	1.720
5 Y	$\beta\,(50\%)$	-0.084	0.057	1.566
	$\%\left(\widehat{\beta}\right)$	**0.020**	0.311	0.569

Table IVB displays coefficients for predictive regressions of excess returns, consumption growth, dividend growth or the risk free rate on predictor variables in the 1930-2008 annual dataset. For consumption growth, dividend growth and excess returns the predictor variables are the risk free rate, the log price-dividend ratio and lagged consumption growth, dividend growth or excess returns. For the risk free rate, the predictor variables are the log price-dividend ratio, consumption growth and the lagged risk free rate. The upper left panel displays the results for excess returns, the upper right for consumption growth, the bottom left for dividend growth and the bottom right for risk free rates. For each coefficients, the median from 100,000 simulations of the BKY model is displayed below. The percentile of the finite sample simulations corresponding to the coefficient is under the median. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Appendix Table V

Moments and the EIS

Moment	*data*	BY $EIS > 1$	BY $EIS < 1$	BKY $EIS > 1$	BKY $EIS < 1$
$E(r_f)$	0.56	2.59	6.08	1.23	NA
$\sigma(r_f)$	2.89	1.21	3.59	0.97	NA
$E(r_m - r_f) + \frac{1}{2}Var(r_m - r_f)$	7.01	5.53	1.96	6.57	NA
$\sigma(r_m - r_f)$	20.46	16.42	12.95	18.57	NA
$E(p-d)$	3.36	3.01	2.92	3.13	NA
$\sigma(p-d)$	0.45	0.18	0.11	0.18	NA

Table V displays moments for the BY and BKY calibrations in annual data for different levels of the EIS. The second column displays the moments for the 1930-2008 annual dataset. The next two columns display the moment for the BY calibration of the model, first for the model with an EIS of 1.5 and then for an alternative model with an EIS of 0.5. The next two columns display the moment for the BKY calibration of the model, first for the model with an EIS of 1.5 and then for the alternative model with an EIS of 0.5. The moments for each combination of preference parameters are medians from 100,000 finite sample simulations of equivalent length to the data. NA refers to cases where the price of a consumption claim is infinite.

Appendix Table VI
Long Run Risks and the EIS: Larger Instrument Set

$$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}$$

Asset	Sample	$\widehat{\psi}$ *data*	$\psi(50\%)$ *BY*	$\psi(50\%)$ *BKY*	$\%(\widehat{\psi})$ *BY*	$\%(\widehat{\psi})$ *BKY*
$r_{f,t+1}$	1930-2008	-0.158	1.642	1.324	**0.000**	**0.017**
	1947.2-2008.4	0.327	1.497	1.360	**0.001**	**0.017**
$r_{m,t+1}$	1930-2008	-0.037	0.033	0.043	0.374	0.257
	1947.2-2008.4	0.023	0.036	0.055	0.479	0.410

$$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right)\Delta c_{t+1} + \eta_{i,t+1}$$

Asset	Sample	$\frac{1}{\widehat{1/\psi}}$ *data*	$\frac{1}{(1/\psi)}(50\%)$ *BY*	$\frac{1}{(1/\psi)}(50\%)$ *BKY*	$\%\left(\frac{1}{\widehat{1/\psi}}\right)$ *BY*	$\%\left(\frac{1}{\widehat{1/\psi}}\right)$ *BKY*
$r_{f,t+1}$	1930-2008	-0.872	1.876	2.107	**0.001**	**0.026**
	1947.2-2008.4	0.625	1.669	1.838	**0.001**	**0.006**
$r_{m,t+1}$	1930-2008	-0.175	0.453	0.324	0.437	0.343
	1947.2-2008.4	0.399	0.500	0.395	0.466	0.503

Table VI displays the EIS estimates using both the risk free rate and the market return as the asset for the BY and BKY calibrations. Medians are from a series of 100,000 samples of equivalent length to the data (948 and 741 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The instruments are consumption growth, the log price-dividend ratio, the risk free rate and the market return, all lagged twice. In the model, the EIS is 1.5. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.